To subscribe, or not to subscribe: Modeling and analysis of service paradigms in cellular markets

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***Abstract*—Traditionally customers subscribe to specific providers and are served by accessing base stations (BSs) of the network of their provider. Inevitably subscribers with relatively "high" usage pattern and data-rate requirements are subsidized by the ones with lower usage and data-rates. As the wireless technology advances, a diverse set of services will be available. This paper introduces the "flex service" paradigm that allows a customer to dynamically access BSs of different providers based on various criteria, such as profile, network conditions, and offered prices. "Flex users" can select the appropriate provider and BS on a per-session basis. This work considers a diverse customer population with respect to their demand, their preference on data-rate over price, their tolerance on the blocking probabilities of their sessions, and their willingness to pay for certain services. Users can dynamically decide to buy a long-term subscription or become flex users. In this paper, we develop a rich framework for modeling and analysis of such markets in different spatio-temporal scales. We analyze the evolution of markets with the flex service paradigm, focusing on whether it can improve the quality-of-service (QoS), social welfare, flexibility and further enhance the competition among providers. The main contribution of this paper is detailed modeling and in-depth performance analysis of such complex markets, in different spatial and temporal scales. It considers the perspective of clients, providers, and regulators. It demonstrates the benefits of markets with the flex service paradigm and compares them with the ones that only offer subscription contracts.**

I. INTRODUCTION

Cellular wireless networks are managed by operators who offer a fixed part of the spectrum to their customers via subscription mechanisms. Subscribers and pre-paid card users are associated with a certain operator to access the spectrum. However new paradigms in wireless access markets and service models are being formed. Unlike the traditional cellular-based markets, these access markets have larger sizes (in terms of number of potential providers), are more heterogeneous (in terms of users and providers), and can offer an improved set of services (e.g., higher multiplexing gains and a reduction of costs due to the higher utilization of existing infrastructure).

As wireless access and use increase, users are differentiated by their usage and data-rate requirement profile. Inevitably subscribers with relatively high usage pattern are subsidized by the ones with lower usage demand. As the wireless network technology advances, a more diverse set of services is made available. To this end, we introduce the paradigm of a "flex user" who is not locked to a specific provider and can dynamically access base stations (BSs) of different infrastructures and providers based on various criteria, such as its profile, the network conditions, and the offered prices. Specifically, flex users are flexible to select the appropriate provider on a *per-session basis*. During a *session*, they transmit and receive data via a BS. This "flex service" paradigm, which has been assumed as a typical access paradigm in wireless LANs, could be a new type of service offered in cellular markets. A similar concept is the "soft" (or virtual) SIM cards.

This work provides a detailed modeling of a cellular market, its providers and a population of users, highlighting the benefits of the flex service and its impact on the evolution of the market. Users could dynamically decide to buy a *long-term subscription* or become *flex users*. As flex users, they can decide about their provider on a per-session basis, while as subscribers they are associated with a specific provider for the entire duration of their contract. The decision making process of a user for selecting the appropriate service paradigm takes into consideration the constraints, demand, and QoS criteria of that user. The terms customers, clients, and users are used interchangeably throughout the paper.

The analysis considers different user populations and the perspective of regulators, users, and providers, possibly with conflicting objectives. An important concern of regulators is to promote competition and social welfare, a fair inclusive treatment of various user populations with respect to services and access. Users target to improve their access (e.g., by reducing their blocking probability) and satisfy their demand, according to their profile, while the revenue maximization is the primary objective of providers.

Several questions drive this research: will this additional service paradigm improve the social welfare by providing more options to users? What is its impact on the revenue of providers and market share? Would the flex service be a viable option for providers and a way to differentiate their services and attain more revenue? How does the traffic demand and user profile shape the decision making mechanism and market share of users? Would subscriptions wane and flex

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users dominate the user population? Does the flex service become the preferable choice for specific groups of users? How are the pricing decisions of the providers being shaped and how do they affect the outcome?

The detailed modeling of the paper allows us to answer the above questions and perform an in-depth performance analysis of a cellular market where providers compete for a *diverse* population of users who can access *these two services*. Two different prices need to be offered for these services. The pricing decisions of the providers are more complicated, but at the same time, allow for a finer partitioning of the market and possibly higher participation for users and revenue for providers. That is, in subscriber-only markets, some users may have been excluded due to the high fee compared to their low willingness-to-pay. In such cases, the flex service can be beneficial thus increasing the overall market pie. Although this is intuitive, other issues are more difficult to be perceived: these markets exhibit several complexities due to the interplay of several parameters (e.g., the competition among providers, and diverse user profiles). While we were analyzing these markets, we realized that their technology and economy aspects manifest different phenomena in multiple spatial and temporal scales. Thus, there is the need for developing a *unified modeling*, analysis and simulation framework that can integrate different scales. To the best of our knowledge, there is no framework in the literature that encompasses all these aspects.

This paper builds on our earlier work [1] and extends it in several ways: it develops a rich modeling framework and simulation platform that allows the analysis of the evolution of access markets at the *microscopic* and *macroscopic* scales. It demonstrates the benefits of the flex service paradigm and offers insights to regulators, users and providers. Another contribution is a preliminary comparative analysis of a simple market at microscopic and macroscopic layers. It also provides new algorithms for setting the subscription and flex rates and considers a new set of metrics, customer types, utility functions, and dynamics.

Section II presents the modeling framework and the simulation platform that instantiates it. Section III focuses on the performance analysis. Specifically, Section III-A describes the cellular-market which was used in our simulations. In Section III-B, we discuss the analysis at the microscopic level focusing on the benefits of the flex service, while Section III-C presents a multi-level analysis of a subscriber-only market. Finally, Section IV overviews the related work and Section V summarizes our conclusions and future work plan.

II. Modeling framework and simulation platform

The wireless access markets integrate networking and business aspects that manifest various spatio-temporal dependencies and localities. For example, user profiles, network topology, and channel conditions may vary across regions. Also, local decisions of users and providers may be required. This motivated us to develop a modeling framework and simulation platform that encompasses microscopic and macroscopic scales. The modeling and simulation of access markets in fine-detail (at the microscopic-level) can be accurate and provide a detailed picture of various phenomena. However, it results in an extremely large number of events, generated by their entities and their interactions. It can be computationally complex to maintain all these events and not amenable to theoretical analysis. On the other hand, macroscopic models of these markets make the theoretical analysis easier but the aggregation results in various inaccuracies.

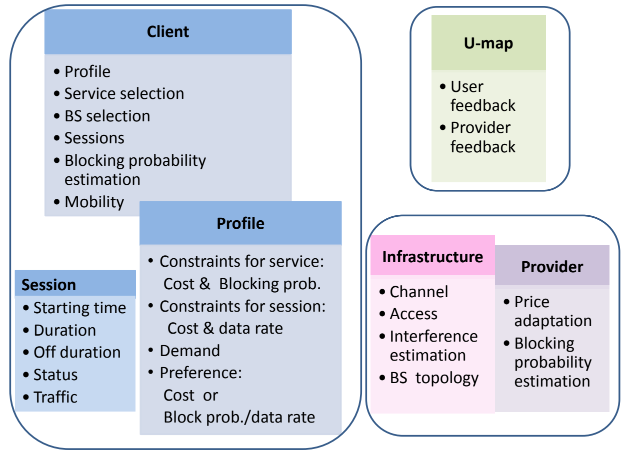

Fig. 1: An overview of the main modules of the modeling framework at the microscopic level.

We are in the process of developing a complete multi-layer *game-theoretical* framework that can instantiate various access markets at both the microscopic and macroscopic levels. So far, we have instantiated subscriber-only and mixed markets (with subscriptions and the flex service) at the microscopic level in the modeling framework and simulation platform. We have also modeled a subscriber-only market at the macroscopic level. These two levels can provide different perspectives and highlight different phenomena but also show the trends that persist in both settings. A detailed description of both levels is provided in the next subsections.

Our modeling framework is fully configurable and parameterized based on the channel, infrastructure and network topology, type of users (e.g., service, demand, mobility, constraints, preferences, decision making processes), providers (e.g., price estimation, services), and available information. We have developed a detailed simulation environment of this framework, which is modular, in that, it can instantiate and implement different models for these parameters.

A. Microscopic level

The main modules of the modeling framework and simulation platform at the microscopic-level include the ones for providers, clients, and u-map (shown in Fig. 1). We analyze the evolution of markets for an extended time duration that corresponds to multiple epochs. Providers offer their subscription and flex services. At the start of each epoch, they set the prices of these services, aiming to maximize their

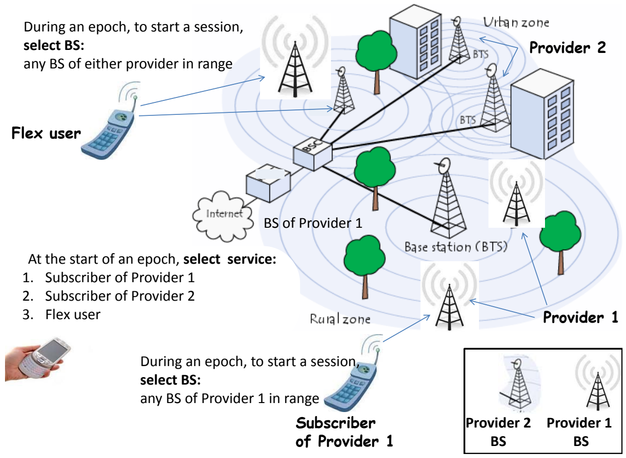

Fig. 2: An example of a cellular-based duopoly with a flex user and a subscriber.

revenue. Clients take two distinct types of decisions, namely a long-term decision for the type of service (subscription or flex) at the start of an epoch, and short-term decisions for the most appropriate BS in a per-session basis (e.g., Fig. 2). Finally, the u-map is a "review" system, which enables clients to upload periodically their feedback about the QoS of their sessions and price of their services. This feedback becomes available through the u-map to other clients as well as providers and is taken into consideration during their decision making. In the following subsections we describe the modules of the modeling framework in more detail.

Infrastructure

Each provider has deployed a cellular topology that offers wireless access via its BSs to clients in a small city. Providers divide their channels into time slots according to a TDMA scheme. To model the channel quality, we employed the *Okumura Hata* path-loss model for small cities considering the contribution of shadowing to the channel gain [2], [3]. The interference power at a BS during a time slot is computed considering the contribution of all interfering devices at co-channel BSs.

Clients

Clients generate requests to connect to a BS to start a *session*. The duration of a session and the *off duration* (i.e., time interval between the end of a session and the start of the immediately next one of the same client) are given by theoretical distributions. Specifically, the *session duration* follows a Pareto distribution, while the *off duration* is generated according to a log-normal distribution [4]. To initiate a session, the client needs to *first* select a BS.

Service types: A client needs to select a *service type* or remain *disconnected*. During a disconnection period, the client does not initiate sessions. Two service types are considered: the subscription and flex service. For a session, a subscriber or flex user needs to select a BS. A flex user may select a BS of *any* provider, while a *subscriber* of a certain provider may select *only* BSs of *that provider*. The service type or the disconnection period lasts for an epoch. The selection of the service type is renewed at the start of each epoch. The BS selection takes place before the start of each session. Note that in our framework the epoch has a fixed duration and lasts for several days (or months). A session lasts for several minutes.

User profile: The profile of a user includes the *constraints, demand*, and *preferences* of that user. The constraints are quantified by four thresholds: two thresholds for the service selection and two thresholds for the BS selection. The constraints of a user (u) for the service selection are expressed by its *willingness-to-pay for a service* ($T(u)$) and its session *blocking probability* tolerance threshold ($B(u)$). The constraints of a user for the BS selection are its *willingness-to-pay for that session* and the *minimum acceptable data-rate*. On the other hand, the preferences indicate the criteria for selecting a service type as well as selecting a BS. These criteria can be either based on the monetary cost of the service or the QoS (e.g., the data-rate, blocking probability).

Service selection: A user selects the service that optimizes the metric that reflects its preference with respect to the blocking probability or the cost. Specifically, in the case of a cost-conscious user, the client selects the service that minimizes its cost spending, while a QoS-conscious user selects the service that minimizes the blocking probability.

BS selection: During an epoch, subscribers and flex users perform sessions. For each session, a client selects a BS in its wireless range based either on the data-rate or price criterion. Specifically, a price-conscious client selects the BS that *minimizes its cost spending*, while a QoS-conscious client selects the BS that *maximizes its achievable data-rate*. For the service selection, a client expresses its preference with respect to cost and blocking probability, while for the BS selection, the preference is over cost and data-rate.

U-map

The market assumes the presence of a *user-centric* data repository that maintains information about the user population. The u-map serves like a simple review/feedback system. In the framework, it is a data structure that corresponds to a grid-based representation of a region. Periodically, each client reports the *percentage of blocked sessions* and its service type at the u-map. Furthermore, each client reports information about its constraints, demand, and preferences as well as the session duration, status, client id, and provider id. Statistics on the mean, median, and maximum blocking probabilities across all subscribers of the same provider and flex users are computed. Providers report their subscription and flex rates at the u-map.

Decision-making of clients

The decision-making process of a client involves *long-term* decisions made at the beginning of each epoch for selecting the service type for that epoch and *short-term* decisions for selecting the appropriate BS at *a per-session basis*. For each

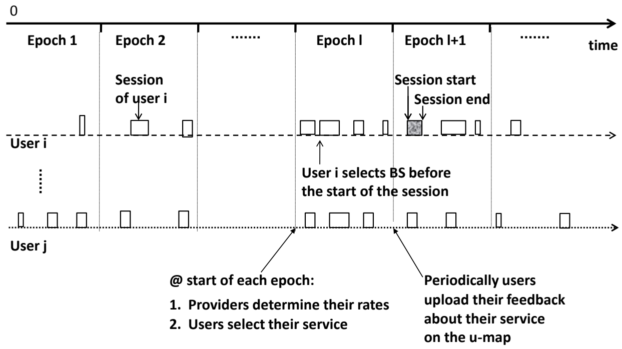

Fig. 3: Decisions of providers and users during the evolution of a market.

of the available service options, namely, to become subscriber of a specific provider, or flex user, *the service constraints need to be satisfied*. Specifically, a client first checks whether the total cost for that service is under its willingness-to-pay threshold (assuming that each client is aware of the average duration of its sessions per epoch) *as well as* the estimated blocking probability of that service is under its blocking probability threshold.

A client needs to estimate the blocking probability for each of the above services. Specifically, the *blocking probability of subscribers of a certain provider* is estimated as the average blocking probability as reported by all subscribers of that provider during the last w epochs at the u-map. The *blocking probability for the flex service* is estimated as the average blocking probability of all flex users during the last w epochs as reported at the u-map.

The user selects the service that optimizes the metric that indicates its preference (i.e., blocking probability or price). Specifically, in the case of a cost-conscious user, the client selects the service that minimizes its cost spending, while for a blocking-conscious user, the client selects the service that minimizes the blocking probability.

After the service-type selection, during an epoch, users perform sessions. For each session, a client selects a BS based either on data-rate or price. Specifically, price-conscious clients select the BS that minimizes their cost spending, while data-rate conscious clients select the BS that maximizes their achievable data-rate. Note that for the service and BS selection, users have predefined preferences. Specifically, for the service selection, the preference is over cost and blocking probability, while for the BS selection, the preference is over cost and data-rate. The decision making of users is shown in Fig. 3. The transmission rate is computed based on the Shannon capacity theorem, although more sophisticated models that take into consideration the modulation schemes can also be easily incorporated [5].

Client and session status: A client becomes *disconnected* when *any* one of its constraints for the *service* selection can *not* be satisfied. Otherwise, the client chooses the service type (subscriber of one of the providers or flex user) that optimizes its objective based on its preference. The status of the client lasts for an epoch and is renewed at the start of each epoch. A session can be *successful* or *blocked*. Specifically, a session is blocked when *any* of the client constraints for the *BS selection* can *not* be satisfied by any BS in the wireless range of the client or *all* the time slots of these BSs have been serving other sessions.

Client mobility: Clients move with pedestrian speed according to the random waypoint model.

Providers

Each provider performs two decision making processes, namely, (a) the estimation of its subscription rate at the *start of each epoch*, and (b) the estimation of the flex rate that takes place *multiple times* during an epoch. The subscription rate is decided at the start of an epoch and remains fixed during that epoch, while the flex rate is updated multiple times during the epoch. A provider does not apply any priority or reservation algorithm for serving the sessions in its cellular infrastructure.

Tariffs/charging schemes: The subscription charging scheme is a two-parameter tariff that includes a *flat-rate* (e.g., p) for an *up to a certain total session duration* (e.g., D_{flat}) and a *fixed per-minute per Watt of transmission power cost* p_0 that charges for any extra session duration. The flat-rate price p is determined at the subscription rate estimation process. For example, the cost of a subscriber u with demand $D(u)$ that buys a subscription with a rate p will be p if $D(u) \leq D_{flat}$, whereas, if its total session duration exceeds the D_{flat} threshold, the subscriber will be charged of $p_0 * \tau * d$, for each extra session of duration d, during which, it invests transmission power of τ. The flex service charging scheme is a simple *linear* tariff which charges the sessions per minute and Watt of transmission power. These pricing schemes that charge the clients proportionally to their transmission power aim to penalize an aggressive increase of the transmission power.

Price adaptation: Providers perform various best-response adaptation algorithms to determine the subscription rates at the start of each epoch and the flex rates multiple times during an epoch. They maintain a macroscopic perspective/knowledge about users by considering *user populations* instead of distinct users. Specifically, they may either consider "representative users", or run a pure macroscopic-level algorithm based on differential equations (as described in the Section II-B). The representative users can be the outcome of a clustering algorithm on the entire user population. Providers run the price adaptation algorithm by simulating *offline* this market.

A novel algorithm based on the concept of representative users is described as follows: first the city is divided into a number of regions $R_i, i = 1, ..., I$. Then the market is simulated for a specific time interval. We denote the set of all sessions that are performed during the experiment as H. A specific session $h \in H$ can be written as a pair (h_u, h_a) where h_u are the characteristics of the user that performs the session (willingness-to-pay, data-rate threshold, and demand) and h_a is the location at which the session is initiated. The characteristics of representative users of a specific region R_i

are determined based on the dataset of sessions that were performed at that region $H(i) = \{h \in H | h_a \in R_i\}$. Specifically, the characteristics of the representative users are determined by applying a clustering algorithm on the dataset $H(i)$ (e.g., the K-means in this paper). To estimate the revenue of a provider at a particular time instance, the price of its competitor is assumed to remain fixed. The positions of the representative users of a region are given by a uniform distribution. The decision making of each representative user is simulated and the revenue of each provider is computed. This process is repeated for all prices that could be offered by the provider. The price that corresponds to the maximum revenue is selected (best response algorithm).

Blocking probability model: A provider *predicts* the blocking probability that will be observed at its network when it offers a subscription rate p_1 given that its competitor offers the subscription rate p_2 during that epoch. The blocking probability is used in the decision making process of the simulated representative users. Specifically, providers employ the following sigmoid function to model the blocking probability:

$$B(p_1, p_2) = \frac{1}{1 + \exp(a_1 p_1 + a_2 p_2 + a_3)} \quad (1)$$

Each provider separately estimates the parameters a_1, a_2, and a_3 using fitting during a training phase. The training takes place dynamically as the market evolves. Specifically, each provider uses the average blocking probabilities for its network recorded at the u-map by its subscribers during a large time period of multiple epochs to fit the parameters a_1, a_2, and a_3. The provider will consider this specific blocking probability $B(p_1, p_2)$ for its subscription rate estimation process.

Subscription rate estimation: Each provider emulates the service selection of all representative users. As mentioned earlier, the constraints of a representative user (namely, its blocking probability and willingness-to-pay thresholds) need to be satisfied. For the estimation of the blocking probability, the provider employs the sigmoid-based model. Then, the provider estimates the expected charge for each representative user based on its demand and for each potential service choice that this user can make (e.g., subscriber with one of the two providers or flex user). Specifically, to compute the extra charge (over the flat rate fee) for the subscription service, the provider considers the average price that all clients of each provider paid during the last epochs. Similarly, the average charge of flex users is computed. Based on the offered subscription rates and the average flex rates, the provider estimates its revenue. After exploring the space of the possible prices, the algorithm reports as the subscription rate of the provider, the price that maximizes its revenue, (given the announced subscription rate of its competitor).

Flex rate estimation: The estimation of the flex rate runs multiple times during an epoch. Each provider estimates its revenue by simulating the BS selection process of each representative user depending on its selected service. In that process, the provider assumes that the flex rate of its competitor remains the same (as in the previous time period). Then each provider chooses the flex rate that maximizes its revenue.

B. Macroscopic level

Unlike the microscopic-level framework that models each entity in relatively fine detail, at the macroscopic-level, a simple model handles users as a homogenous population with respect to preferences and decision-making mechanism. More specifically, in such a model, the behavior of users is described by a population game that determines how the entire user population reacts to the decisions of providers. For example, when the game is at an equilibrium and a provider changes its strategy (i.e., offered price for a service), the population game models the resulting *flow* of users among the offered services until the system reaches again a new equilibrium. The competition of providers can be also modeled as a game in which different sets of offered prices result in different equilibrium points for the user population, and by extension, to different levels of revenue for providers. Table I describes the parameters of the modeling framework at the macroscopic level.

TABLE I: Parameters of the macroscopic model

Parameter	Description
t	Time
r	Speed of user dynamics
$z(t)$	Distribution of users to strategies at time t
$U_i(z(t))$	Utility of users that choose strategy i
ϵ	Noise parameter of Logit dynamics
p_i	Subscription rate of provider i
b	Price sensitivity of users
$-k$	Utility of disconnection state
$q(q_i)$	Probability of a subscriber (of provider i respectively) to be able to achieve its target rate
N_i	Average number of users in a cell of provider i
$M_i(t)$	Average number of subscribers of provider i in one of its cells at time t
$m(m_i)$	Time slots available at BS (of provider i respectively)
μ	Session death rate
λ	Session birth rate

We instantiate the aforementioned game of a subscriber-only market with two providers that compete by setting the prices to maximize their own revenue. Clients decide based on the offered prices and the blocking probability. These prices are flat rates and are determined at the beginning of each epoch.

We consider the same cellular infrastructure as the one at the microscopic level. [1] Moreover, we assume that the traffic demand of each subscriber is modeled by an on-off process with exponential on and off periods of rates μ and λ, respectively. Based on this assumption, each cell can be viewed as an M/M/m/M queue [6], where m is the number of time slots that are available at the BS and M is the number of clients that compete for access to these slots (i.e., subscribers in that cell). In an M/M/m/M queue, the session arrival rate is proportional to the number of subscribers that are not performing sessions. No more than m subscribers are

[1] The interference is computed based on the assumption that there are interfering users at all six closest co-channel cells and their positions are at the centers of these cells.

allowed in the system, that is, a session will be immediately blocked if there are no available slots at the BS (Fig. 4).

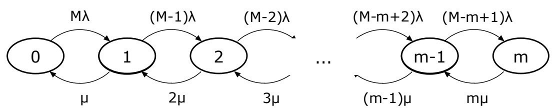

Fig. 4: The M/M/m/M queue

Blocking probability model: The blocking probability is given by the Engset formula (Eq. 2). Finally, due to the assumption of uniform user spatial distribution, the blocking probability at an arbitrary cell can be estimated, and then, generalized for the entire network. [2]

$$P(M) = \frac{\binom{M}{m}\left(\frac{\lambda}{\mu}\right)^m}{\sum_{j=0}^{m}\binom{M}{j}\left(\frac{\lambda}{\mu}\right)^j} \tag{2}$$

We also assume that clients aim to achieve a transmission rate that is strictly larger than a *target-rate threshold*. Due to the symmetry of the channel model, we can compute the maximum distance (parameter l) from the BS, in which clients can achieve their target-rate and the probability of a subscriber to be within distance l from the BS (parameter q). Based on the above, the expected blocking probability is defined as: $B(M) = 1 - q + qP^c(qM)$. The variable M in Eq. 2 is a natural number, while the parameter qM can take real values. To resolve this we define a new function $P^c(x)$ which is continuous and is estimated by performing a cubic spline interpolation [7] on the discrete function $P(M)$.

Definition of the user dynamics: To estimate the market share, we first define a vector $z(t) = (z_1(t), z_2(t), z_3(t))$ that indicates the probabilities with which a user chooses to become subscriber of the provider 1 or provider 2 or remain disconnected at time t. The blocking probability for the network of the provider i can be written as:

$$B_i(z_i(t)) = 1 - q_i + q_i P_i^c(q_i z_i(t) N_i) \tag{3}$$

where $z_i(t) = M_i(t)/N_i$, N_i is the average number of users in an arbitrary cell of the provider i and $M_i(t)$ is the number of subscribers of the provider i in that cell, at time t.

The user utility can be defined based on the blocking probability that is associated with a certain service (Eq. 3), the price of that service, and the significance with respect to price (indicated by the weight b).

$$U_i(z(t)) = \begin{cases} -B_i(z_i(t)) - bp_i & \text{if } i = 1,2 \\ -k & \text{if } i = 3 \end{cases} \tag{4}$$

The term $-k$ is a threshold under which the user prefers to remain disconnected. Using the utility function of Eq. 4, the user dynamics can be defined based on the Logit dynamics, as a system of ordinary differential equations (5).

[2]This procedure can be extended for non-uniform user spatial distributions.

$$\begin{aligned} \frac{dz_1(t)}{dt} &= r * \left(\frac{1}{1+F_1(z(t))} - z_1(t)\right) \\ \frac{dz_2(t)}{dt} &= r * \left(\frac{1}{1+F_2(z(t))} - z_2(t)\right) \end{aligned} \tag{5}$$

The parameter r controls the speed of the dynamics, while the functions $F_1(z(t))$ and $F_2(z(t))$ are defined in Eq. 6.

$$\begin{aligned} F_1(z(t)) &= \exp\left(\frac{U_2(z(t))-U_1(z(t))}{\epsilon}\right) + \exp\left(\frac{U_3(z(t))-U_1(z(t))}{\epsilon}\right) \\ F_2(z(t)) &= \exp\left(\frac{U_1(z(t))-U_2(z(t))}{\epsilon}\right) + \exp\left(\frac{U_3(z(t))-U_2(z(t))}{\epsilon}\right) \end{aligned} \tag{6}$$

The term ϵ is a noise parameter. When ϵ tends to infinity, users tend to select a strategy based on a uniform probability distribution. In such a case, the system converges to the equilibrium $(1/3, 1/3, 1/3)$ which corresponds to equal probability for choosing each of the available strategies. When ϵ tends to zero, users tend to select the strategy that maximizes their utility function at the current state.

The providers determine the subscription rates at each epoch based on the best response algorithm. Specifically, at the beginning of each epoch, the providers take actions simultaneously. A provider offers the price that maximizes its immediate revenue, assuming that the price of its competitor will not change. The revenue is computed by integrating the instantaneous revenue for the entire duration of the epoch. The above procedure is repeated at each epoch until the system converges to a Nash equilibrium.

III. Performance Evaluation

A. Simulation setup

We implemented the microscopic and macroscopic modeling framework in Matlab and performed Monte Carlo simulations for different market scenarios. The preferences and constraints of a user remained *fixed throughout the simulation scenario*. Each run represents the evolution of the market during a period of 50 epochs, each lasting 5 days. This long duration is required in order to better observe the evolution of providers, their interaction with users in this simulated small-city environment, and identify transient and steady-state phenomena. For the service selection of users, the blocking probability was estimated based on data that correspond to the last 10 epochs as reported by clients on the u-map.

Wireless network infrastructure: The simulation platform corresponds to a small city, represented by a grid of 11 km x 9 km. Each provider has a cellular network that consists of 49 BSs placed on the sites of a triangular grid, with a distance between two neighboring sites of 1.6 km. This setting corresponds to a typical microcellular network deployed in a small city [8]. Moreover, each provider owns bandwidth of 5.6 MHz, that is divided into 28 channels of 0.2 MHz width. These channels are allocated to BSs according to a frequency reuse scheme with spatial reuse factors of 4 and 7, for provider 1 and provider 2, respectively. Each channel is further divided into three time slots in a TDMA scheme, resulting in 21 time slots per BS of provider 1 and 12 slots per BS of provider 2. A single time slot of a given BS can be offered to *only one* *client*. Each client is associated with one BS during a given session. The maximum allowable transmission power that a client can invest is 2 Watts.

User population: There are 28000 users in total, distributed according to a uniform distribution in the simulated region of this small city.

B. Simulations and analysis at the microscopic level

User profile: The constraints of users, namely their willingness-to-pay and blocking probability thresholds for the service selection as well as their data-rate and price tolerance thresholds for the BS selection, follow Gaussian distributions (their parameters are shown in Table II). The name convention "X-Y" indicates with "X" the service selection criterion and with "Y" the BS selection criterion (as shown in Table III). In each scenario, *all* users use the same criterion for their service and BS selection.

TABLE II: **User thresholds follow Gaussian distributions**

Threshold	Mean	Std
Willingness-to-pay (cost/min): Service selection	0.170	0.037
Blocking probability	0.200	0.050
Willingness-to-pay (cost/min): BS selection	0.150	0.037
Data-rate (Mbps)	0.100	0.010

Client demand: A client generates a sequence of session requests. The session duration follows a Pareto distribution ($x_s = 3.890$, $a = 4.500$) of mean 5 min. The disconnection period follows a log-normal distribution with different parameters for each user: a location parameter uniformly distributed in the interval $[4.068\ 6.215]$ and a scale parameter equal to 0.368. This corresponds to a client demand that varies from 33 to 267 minutes (in total) per epoch. During disconnection periods, clients move with pedestrian speed of maximum value 1 m/s, while they remain stationary during sessions. Furthermore, during a session, the client remains connected at the same BS for the entire duration of the session.

TABLE III: **Simulated scenarios**

Scenario	Service criterion	BS criterion
B-R	Blocking probability	Data-rate
B-P	Blocking probability	Price
P-R	Price	Data-rate
P-P	Price	Price

To highlight the impact of the flex service, two market types were simulated: a *subscriber-only market* (baseline case), in which each user has only the choice of becoming a subscriber of one of the providers or remain disconnected, and a *mixed market*, in which users have the additional service option of becoming flex users. The analysis evaluates the impact of service paradigms on the evolution of the market, using metrics that can provide insights to regulators, users, and providers.

The performance of a provider is characterized by its revenue, while the performance of a client is indicated by the blocking probability of its sessions. Furthermore, we quantify the overall satisfaction of the society by computing the session blocking probability, social welfare, market share, and percentage of disconnected users. The *session blocking probability of a client* is the ratio of its blocked sessions over the total number of session requests. The *social welfare* is defined as the sum of the net benefit of all users and providers. The net benefit of a provider is its revenue, while the net benefit of a user is the difference of what the user was willing to pay and what the user actually paid for his/her sessions. Our reported results are average statistics over all epochs and Monte Carlo runs. Let us now discuss the main results of our analysis.

Benefits of the flex service: We comparatively analyze the subscriber-only and mixed markets at the microscopic level. Indeed, the flex service becomes a catalyst in the market! In most scenarios, there is a dramatic decline of the number of disconnected users, a prominent reduction in the blocking probability, and an increase in the social welfare (as shown in Figs. 5a, 5b, and 5c, respectively). Interestingly, P-R only exhibits a slight decrease in the percentage of disconnected users and blocking probability in the mixed market compared to the subscriber-only one. This is due to relatively high subscription and high flex rates (Figs. 6c and 6f, respectively) which surpass the willingness-to-pay of many users. [3] The flex service reduces the competition between providers resulting in slightly higher subscription rates compared to the ones in the subscriber-only market. Specifically, each provider does not intend to reduce its subscription rate below a certain threshold, because by doing so, it will lose profit from flex users. This results in increased revenue for both providers and increased percentage of disconnected users and blocking probability.

Note that due to the larger participation in the mixed market compared to the subscriber-only one, the social welfare is improved. Moreover, flex users exhibit significantly lower blocking probabilities than subscribers in all scenarios.

The effect of user preference: User decisions in the market are affected both by the price and the QoS. For example, in B-R, the driving force of the user preference is the QoS, while in P-P, the cost of the service. When the price is the primary driver in the user selection, the revenue of the providers is constrained. On the other hand, in B-R, the providers have more flexibility to set their prices and achieve larger revenue. These markets are somewhat extreme cases. The B-P and P-R markets integrate both the price and QoS requirements in the user selection process. The essential difference between B-P and P-R is that the first applies the QoS preference at the service selection, while the second, at the BS selection. In general, the scenarios can be ordered with respect to the achieved revenue of providers as follows: B-R $>$ B-P $>$ P-P. The same order holds when compared with respect to the percentage of disconnected users.

Gains due to channel availability and quality: Both the channel availability and the channel quality affect the rev-

[3]Note that in the mixed market, B-P exhibits higher subscription rates than P-R. However in B-P, the blocking probability and percentage of disconnected users are relatively low. The relatively low flex rates attract most users and the subscription rates do not affect the market.

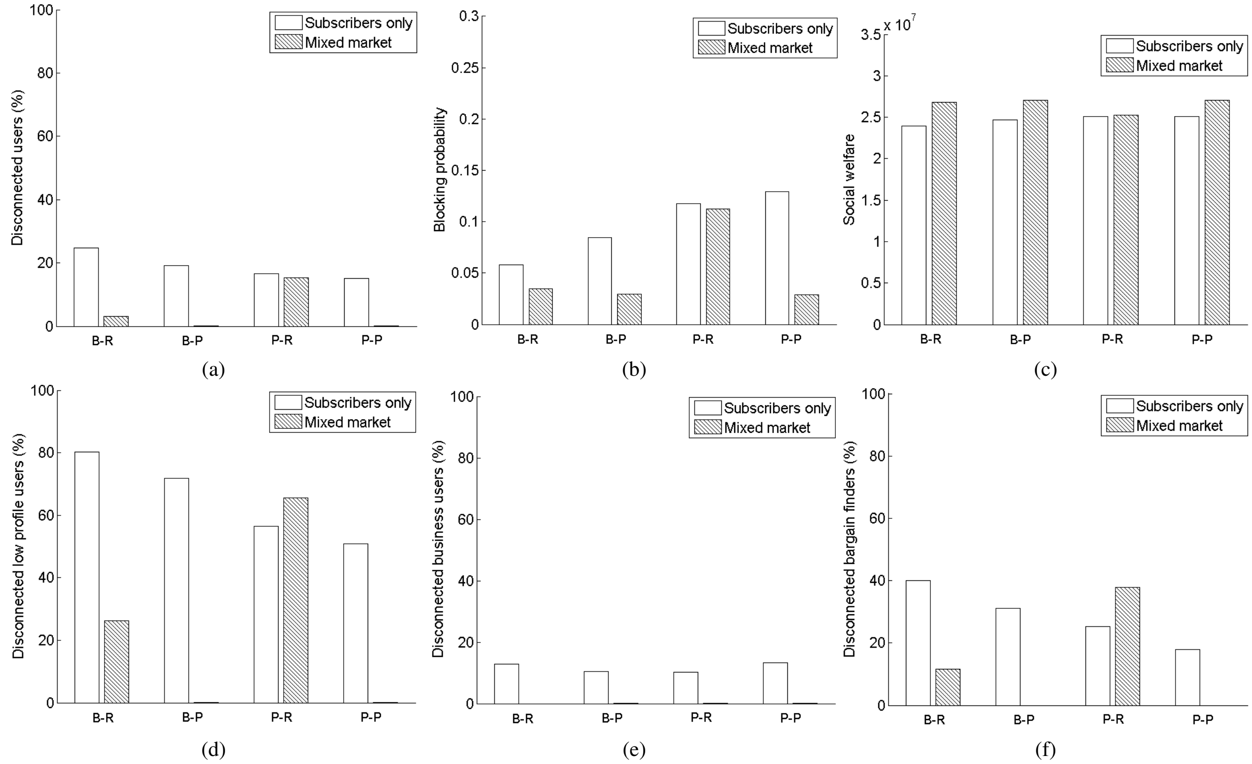

Fig. 5: (a) Percentage of disconnected users. (b) Blocking probability. (c) Social welfare. (d), (e), and (f) Percentage of disconnected low-profile, high-business, and bargain-finder users, respectively.

enues of the providers. However depending on the user traffic demand, their significance changes. The performance of providers is coupled with the blocking probability of their networks which strongly depends on the number of available slots at BSs. Under large user population, the number of slots gives an advantage to the provider 1 in terms of market share and revenue. Furthermore, the provider 1 can afford to offer a higher subscription rate compared to its competitor, "boosting" its revenue. However, under small user population (e.g., 5000 users), the channel quality becomes the decisive factor in the user selection process. In such a case, the provider 2 gains the advantage. Note that on average, the provider 2 offers channels of better quality compared to the provider 1 due to the higher spatial reuse factor of its network.

The impact on different user populations: We distinguished three user populations, namely the *high-business, bargain-finder*, and *low-profile users* (Table IV) and observed their performance in the context of the two markets. The flex service affects mainly the low-profile and high-business users. Specifically, high-business users with very low blocking probability threshold end up selecting the flex service due to its lower blocking probability compared to subscriptions. On the other hand, low-profile users with very low traffic demand (i.e., few sessions per epoch) also select the flex service because its cost is lower compared to subscriptions, even if the cost of each session is high. However, the flex rates play an important role on the share of flex users. Specifically, when the flex rates are low, a significant percentage of flex users are high-business users (especially at the B-R and B-P scenarios). On the contrary, when the flex rates are high, flex users are mostly low-profile users. This is due to the fact that when flex rates exceed the user willingness-to-pay threshold, flex users will experience an increased blocking probability. Actually, in these cases, the flex service exhibits higher blocking probability than subscriptions, which explains the relatively lower percentage of high-business users.

TABLE IV: **User Populations**

Type	Willingness-to-pay	Blocking Probability	Demand
High-business	$>$ 80% percentile	$<$ 20% percentile	all range
Bargain-finders	$<$ 20% percentile	$>$ 80% percentile	all range
Low-profile	$<$ 50% percentile	all range	$<$ 20% percentile

On the exclusion effect: From the perspective of regulators, this is an important implication to the social welfare. To highlight how a subscriber-only market excludes certain user populations (e.g., the low-profile users), the percentage of disconnected users was computed: we found that in B-R, it drops from 80% in the subscriber-only market to 26% in the mixed market (Fig 5d). On the contrary, in P-R, the percentage of the disconnected bargain finders and low-profile users increases in the mixed market compared to the subscriber-only one. As mentioned earlier, this is due to the increased subscription and flex rates in the mixed market (Fig. 6c, 6f), which surpass the willingness-to-pay threshold of a larger number of users compared to the subscriber-only market. In B-P and P-P, the percentage of disconnected low-profile users drops substantially (Fig. 5d). Moreover, the percentage of the

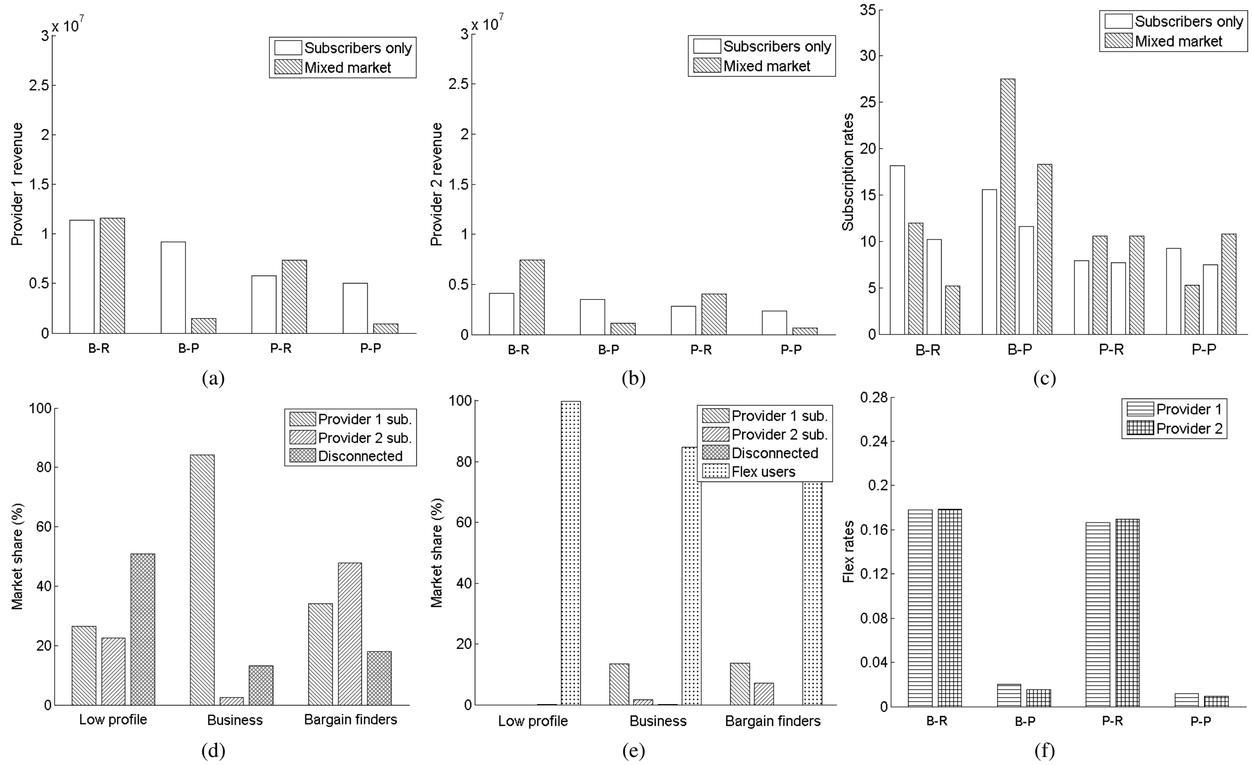

Fig. 6: (a) The revenue of provider 1. (b) The revenue of provider 2. (c) The subscription rates. For each scenario, the left column corresponds to the provider 1 and the right column to the provider 2 of that market, respectively. (d) Market share in the subscriber-only market for the P-P scenario. (e) Market share in the mixed market for the P-P scenario. (f) Flex rates in the mixed market.

high-business disconnected users is close to 0 in all scenarios (Fig. 5e)! The percentage of disconnected users of other user profiles are shown in Figs. 5d, 5e, and 5f.

Effect of the averaging window: We considered different windows of time for the estimation of the average blocking probability. The smaller the window, the larger the intensity of the oscillations in the market share, revenue, and blocking probability. Often, a large portion of users moves between providers. Note that, the user feedback in the u-map becomes available to other users after a certain period of time. This delay contributes in oscillations on the market share, and subsequently, on the blocking probability. Thus, the oscillations caused by the price competition of providers are enhanced.

Service selection based on blocking probability: For the BS selection, clients select the BS based on the data-rate or price criterion. The subscriber-only market shows high percentages of disconnected users and relatively high blocking probabilities. In general, due to the higher channel availability of the provider 1 compared to the provider 2, that directly affects the observed blocking probabilities, the revenue of the provider 1 is larger. This difference is prominent especially in B-R (Fig. 6a, 6b).

Interestingly, and not necessarily expected, in the mixed market with rate preference (B-R in Figs. 6a and 6b), not only the percentage of disconnected users and the blocking probability have decreased but also the revenue of the provider 2 has increased substantially. On the other hand, the revenue of the provider 1 is almost unaffected. This means that the prices allow users to select the most suitable product depending on their demand (expected number of sessions), which results in the increased user participation. Subscribers tend to select the provider 1 that has the lowest blocking probability (due to its larger channel availability). Although the subscription rate of the provider 2 is significantly smaller compared to the provider 1 (Fig. 6c), the blocking probability criterion for the service selection gives a distinct advantage to the provider 1.

The flex service is preferred by users sensitive to blocking probability and data-rate and with higher willingness-to-pay threshold, which results in relatively higher flex rates. All high-business users in the mixed market choose the flex service, given that it offers the lowest blocking probability.

In B-R, in both markets, the provider 2 attracts more bargain-finder and low-profile users than the provider 1 due to its lower subscription rate: even if the blocking probability of the provider 1 is lower compared to the provider 2, its subscription rate is higher and cannot satisfy the willingness-to-pay threshold of the majority of low-profile and bargain-finder users. These users have no other choice but to become subscribers of the provider 2. In addition, the percentage of flex users from these populations is significantly lower than that of high-business users, given the relatively higher rates of the flex service. In B-P, the difference in the percentage of disconnected users of the two markets is even more dramatic. Similar reductions are observed in the per user and per session blocking probabilities. Moreover, the price criterion intensifies the competition, which has as a result, a more prominent reduction in the offered prices, causing a steep decrease in the revenue of the provider 1 (Fig. 6a). Even more interestingly, the population of subscribers dies out! Note that the price preference at the BS selection affects dramatically the flex rates: the competition between providers lowers the flex rates which encourages all users to become flex users.

Service selection based on price: The price criterion in the service selection weakens the advantage of the low blocking probability of the provider 1 resulting in a prominent reduction of its revenue (Fig. 6a). In addition, the competition of the providers in the offered subscription and flex rates encourages more users to become subscribers compared to the markets in which the blocking probability was the service selection criterion. The preference of users in lower price over blocking probability results in an increase of the blocking probability.

Flex rates: A general trend in the mixed market is that the difference of the flex rates between providers is very small (as shown in Fig. 6f). This can be explained by the symmetry in the deployments of the two providers and the uniform distribution of clients in the region. As mentioned earlier, the flex rates are determined and are affected by the BS selection mechanism, in which the position of the clients and the BS deployment play an important role. The flex service product market is actually a commodity market with an almost identical "market price" across all competing providers (same with price for any kind of commodity goods ranging from crude oil to Internet transit prices) [9]. In the case of rate preference, in the BS selection, the flex rates are relatively increased, compared to the subscription rates. On the other hand, in price preference, the price criterion forces the providers to keep their flex rates at relatively lower levels.

Subscription rates: Notice that in most scenarios, the average subscription rate of the provider 2 is lower compared to the provider 1 (as shown in Fig. 6c). Moreover, the differences between their subscription rates is larger in B-R and B-P compared to P-R and P-P. This is due to the advantage of the provider 1 in the blocking probability which significantly affects the client decisions in B-R and B-P. On the contrary, in P-R and P-P, price is the parameter that mostly affects the client decisions resulting in small differences between the subscription rates.

An exception to this trend occurs in the mixed market in P-P. There, provider 2 offers regularly a very high subscription rate (on average higher compared to provider 1). This is due to the sigmoid model which makes the provider 2 to believe that if it increases its subscription rate, it will improve the blocking probability of the flex service. Specifically, in some cases, the provider 2 cannot achieve additional revenue by lowering its subscription rate. If the sigmoid model predicts a decrease in the blocking probability of the flex service by increasing the subscription rate, the provider 2 will increase its subscription rate (aiming for higher revenue from flex users). However, this choice of the provider 2 does not significantly affect the blocking probabilities and revenue. This is an example of cases in which the sigmoid model misleads a provider to take a non-profitable decision.

C. Simulations of the macroscopic level and comparative multi-level analysis

To better explore and highlight the benefits of the multi-layer framework, we then implemented the macroscopic modeling framework (described in Sec. II-B) and simulated the subscriber-only market at the macroscopic level. To enable the comparison between the microscopic and macroscopic models, we implemented also a subscriber-only market at the microscopic level that uses the same user utility function and price-adaptation algorithm as the one at the macroscopic level. Specifically, in this new microscopic-level market each provider runs the following algorithm to adapt its subscription rate: at the beginning of each epoch, it emulates the macroscopic level offline and chooses the rate that maximizes its revenue based on the assumption that its competitor will "stick" to the price it offered during the previous epoch. Finally, the user profiles are determined based on the parameters k and b (Table I) which follow Gaussian distributions with a mean of 0.800 and 0.025, respectively, and a standard deviation of 0.100 and 0.005, respectively. This microscopic market becomes closer to the macroscopic one, making their comparison easier. Furthermore, by comparing these instances of a subscriber-only market, we can highlight the common trends and differences.

Several trends persist at both the microscopic and macroscopic levels. For example, the providers exhibit similar behavior: the provider 1 has the advantage in terms of market share and revenue due to its larger channel availability that results in lower blocking probability for its subscribers. The mean values of the revenue, blocking probability and market share of the microscopic-level market are similar to their corresponding ones at the macroscopic-level. Furthermore, the provider 1 tends to offer a higher subscription rate compared to that of the provider 2. At both the microscopic and macroscopic levels, when the price sensitivity of users increases (parameter b in Eq. 4), the subscription rates and revenue of providers decrease. Moreover, the price sensitivity of users "forces" providers to keep the subscription rates at relatively low levels. In addition, this prevents the occurrence of high oscillations of the subscription rates.

The analysis reveals also several interesting and unexpected outcomes: surprisingly, while the blocking probability, market share, and revenue are very smooth at the macroscopic level, they exhibit intense oscillations at the microscopic level. At the microscopic level (as described in Section II-A), clients upload their feedback at the u-map periodically, and thus, this feedback becomes available to the other participants after a certain time. As the uploading period of clients increases, the market oscillations become more intense. On the other hand, at the macroscopic level, this feedback is "disseminated instantaneously" (as expressed in Eq. 5). We speculated that the delay of disseminating the user feedback about the perceived performance is the cause of the oscillations in the market (e.g., in the market share, subscription rates). We validated our hypothesis by simulating the macroscopic level with a time-delay parameter in the user dynamics.

We can explain this phenomenon as follows: assume that in absence of delay, the market share converges to the stable equilibrium point z'. Now consider the case of a time lag τ in the user feedback dissemination. To visualize it, consider the trajectory of the market share evolution. At first the market share increases and reaches the equilibrium z' at time t_1. Due to the time lag, the increase of the market share will continue until time $t_1+\tau$. At this point, users will observe the equilibrium conditions and as such the market share will not increase any further. Then, after a small period of time, users will realize that the market share is above z'. This will trigger a decrease in the market share until the system reaches again z' at time t_2 now from above. The decrease of the market share will not stop until time $t_2+\tau$. This procedure is then repeated and causes oscillations in the market. [4] As mentioned in Sec. III-B, another phenomenon that affects the market oscillations is the averaging window w (i.e., the number of last epochs based on which the blocking probability is estimated). As w increases the estimated blocking probability does not change very rapidly from epoch to epoch and the market oscillations are smoothed out. This is related to some open questions in decision making of stochastic control systems with game theoretical aspects. To the best of our knowledge, the impact of time lag and averaging window have not been considered or analyzed in wireless services/access markets before. Most related studies have focused on competition of providers.

Our extensive simulations of the microscopic level models indicate that several trends persist even for different utility functions, user profiles, and price setting algorithms. In these sets of experiments, under large user population, the provider 1 maintains the advantage in terms of revenue, and market share, and its subscribers achieve a lower blocking probability, while the opposite trend occurs under small user population. However, there is an important difference between these models, namely, under the linear user utility function, the time series of the subscription rates are very smooth and usually converge to an equilibrium set of prices or a periodic solution, while in case of user profile based on thresholds, these time series are more bursty due to the sigmoid-based model.

IV. Related work

A part of the game-theoretical research on spectrum access markets explores competitive pricing (e.g., [12], [13]), while other papers consider providers that cooperate to maximize a common utility function (e.g., [14]). The available information is another critical parameter. For example, some studies consider complete knowledge, i.e., each entity knows the utility functions and the strategies of all others (e.g., [13], [14]), while other studies assume partial knowledge [15]. In general, pricing issues in spectrum access markets have received considerable attention in the literature. However, most of the papers have addressed these issues in a somewhat narrow scope and with simplistic models. Al Daoud *et al.* [16] consider a single provider of a CDMA-based cellular wireless network, who leases to a single lessee a part of his network, thus attaining revenue from both the leased part and the remaining part. Both this paper and the earlier work [17] lack a realistic model of the clients and their demand; sessions and clients are modeled as identical and client mobility is not taken into account. Most importantly, these models do not allow for users to choose a certain provider given the price policy employed; instead an "exogenous" function is used to depict demand. Overall, this paper and many similar ones, though providing some insight to the pricing issue, fail to provide a complete and realistic model of the market and the network as a whole, including user incentives and behavior.

Paschalidis and Liu [18] establish an asymptotically optimal static pricing scheme for sessions, assuming that traffic load and system capacity tend to infinity. Their assumptions are somewhat restrictive for the currently underutilized cellular networks. Unlike that paper, our work focuses on the evolution of the market, highlighting various complex dynamics and transient phenomena, and aims to provide insights for a multitude of settings regarding the user population, preferences, mobility, demand, and network load. Maillè *et al.* [19] derive a model where two providers compete for end users and one of the networks is also a client of the other since it purchases a part of its spectrum. They investigate under which conditions the leasing of spectrum is beneficial for the primary provider and analyze various market phenomena. However, their assumption that the secondary provider is by definition more efficient in terms of spectrum usage compared to the primary is not necessarily realistic. In addition, their microscopic layer modeling does not consider client distributions, mobility, and spatial reuse. The agent-based economies and pricing issues is the focus of [20] (part of the infoecon activity [21]), which provides insights about several interesting phenomena, such as price wars, niche markets, market oscillations. In our work, we also investigate and explain such phenomena as the proposed cellular markets evolve. Note that claims on optimality made on various related papers (e.g. [16], [22], [19]), with respect to the revenue of providers, the social welfare, and market efficiency, though valid in their context, in which providers compete for selling one good for a certain price, may not hold in our setting. In general, product differentiation allows for a finer market segmentation and can further increase the revenue of providers.

Another aspect in the related work is the level of detail (scale). Typically researchers have been modeling a spectrum access paradigm either at the microscopic level or at the macroscopic level. The microscopic-level models consider the interactions among all participating entities in a market, at a very fine level of detail, mostly assuming a limited number of such entities due to the high computational complexity (e.g.,

[4] Hutchinson and Ruan [10], [11] had observed similar phenomena in their models of biological systems.

[23], [24], [14]). Unlike them, the macroscopic-level models consider only an "average" behavior of certain types of entities (e.g. users) to make the analysis more tractable (e.g., [25], [26], [12]). In contrast to these approaches, this paper sets the foundation for developing a complete multi-layer framework that allows the instantiation of an access market at both the microscopic and macroscopic scales.

V. Conclusions and future work

To analyze access markets, we developed a modular multi-layer modeling framework and simulation platform that takes into consideration a diverse set of user profiles and various performance metrics. This paper shows that the flex service is an attractive option in the cellular market that can provide significant benefits compared to traditional subscriber-only markets. The analysis demonstrates the following trends: the flex service dramatically reduces the percentage of disconnected users, decreases the blocking probabilities, and improves the social welfare. The merits are prominent also for specific user populations, such as high-business, bargain-finders, and low-profile users. A user can select the most suitable product that matches its profile, thus increasing participation in the market. In cases in which the user population is sensitive to blocking probability and data-rate, the flex service can improve the revenue of providers. Furthermore, even under different utility functions and price-setting algorithms, several trends persist: for large user populations, the provider with most resources (i.e., slots) outperforms in terms of revenue and market share. The reverse trend holds for small user populations. This difference between providers is reduced in cost-driven markets compared to QoS-driven ones.

Throughout our experiments, at the microscopic level, strong oscillations were observed. These oscillations were caused by the competition among providers as well as the delay in the dissemination of the user feedback via the u-map. The presence of multiple services may also enhance the competition. Depending on the user preference, the flex service may suppress or increase the prices (revenue). In some cases, the myopically greedy pricing results in suboptimal performance and pathologies triggered by the feedback loops caused by the blocking probability estimation model. In general, the flex service allows a finer market segmentation offering more options to providers and users. Providers can also employ more sophisticated pricing schemes that take into consideration the user needs to increase their revenue.

The modeling at different scales pays off: the comparative analysis of subscriber-only markets allows us to generalize several results: e.g., the mean values of revenue, market share, and blocking probability are similar in the two scales. It also allows us to assess the impact of the delay in the user-feedback dissemination and the availability of information across users on the evolution of the market. It is part of our ongoing research to include the flex service at the macroscopic layer.

The collaboration with industrial partners in instantiating markets of interest, customer populations, pricing methodologies and various business-driven aspects is certainly desirable. One of our research directions is to consider a larger number of providers, possibly with different types of information, long-term objectives, trunk-reservation policies, and coalition strategies. We plan to extend the modeling framework and simulation platform to incorporate these aspects. This paper sets the foundations for modeling and analyzing such markets.

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