Dirichlet-based Dynamic Movement Primitives for encoding periodic motions with predefined accuracy

Dimitrios Papageorgiou¹, Despina Ekaterini ARGiropoulos² and Zoe Doulgeri¹

Abstract—In this work, the utilization of Dirichlet (periodic sinc) base functions in DMPs for encoding periodic motions is proposed. By utilizing such kernels, we are able to analytically compute the minimum required number of kernels based only on the predefined accuracy, which is a hyperparameter that can be intuitively selected. The computation of the minimum required number of kernels is based on the frequency content of the demonstrated motion. The learning procedure essentially consists of the sampling of the demonstrated trajectory. The approach is validated through simulations and experiments with the KUKA LWR4+ robot, which show that utilizing the automatically calculated number of basis functions, the predefined accuracy is achieved by the proposed DMP model.

I. INTRODUCTION

The motions performed in robotic applications can be divided in two categories, the non-periodic (discrete) motions and the periodic (rhythmic) ones. Some examples of non-periodic motions are the pick-and-place task or the hand-over of an object, while some examples of periodic motions are the gait, the wiping of a surface, the repetitive execution of a task or the shake-motion of a handshake. Over the past decades, industrial robots are programmed explicitly, by splitting the motion in a sequence of simple motion segments between key frames. The explicit programming of a robot is time and effort consuming and requires technical knowledge. The amount of effort and time required for the explicit programming of a robot is increased even more in the case of periodic motions [1]. For instance, imagine the difficulty involved in the explicit programming of the gait motion of a humanoid robot in its joint space.

Learning from Demonstration (LfD) was recently proposed as a replacement of the classical robot programming. Unlike explicit programming, LfD can be performed easily and intuitively by anyone without requiring technical skills. According to LfD, the motion is taught to the robot based on demonstrations. One key feature of LfD is its ability to generalize the learned motion in space and time during the autonomous execution, by appropriate scaling. The most popular method for encoding the demonstrated motion in LfD is the utilization of dynamical systems for generating the motion. To encode a motion via a dynamical system, the values of its parameters have to be calculated, such that its evolution optimally reproduce the demonstrated motion. In most cases, such dynamical systems employ function approximation techniques, utilizing a weighted sum of kernel functions.

The most popular dynamical systems for encoding periodic motions in the literature are the Dynamic Movement Primitives (DMP) [2], [3], [4], which are widely utilized in many applications, e.g. in human-robot collaboration [5], progressive automation [6], force control [7]. One property of the DMPs is that they can be trained based on a single demonstration. In their classical formulation, DMPs utilize the Gaussian-like von Mises kernel, while the reproduction accuracy depends on the number of kernels utilized [2]. However, one cannot predetermine the minimum required number of kernels for a motion with a given complexity in order to reach a predefined accuracy, which means that if such requirement exists, a trial-and-error procedure should be performed, that is time consuming. Recently, a Probabilistic Movement primitives (ProMPs) variant was introduced based on Fourier decomposition, which does not involve the manual tuning of any hyperparameter such as the number of kernels [1]. However, ProMPs cannot learn a motion based only on a single demonstration and therefore the notion of reproducibility accuracy is not applicable.

In this work, we address the encoding of periodic motions, based on a single demonstration, utilizing the periodic sinc kernels, the so called Dirichlet kernels. The utilization of sinc base functions was introduced in [8] for DMPs, in order to encode non-periodic point-to-point motions. However, the energy of the demonstrated signal in periodic motions is not finite, as opposed to the discrete motions, thus it is not possible to apply the method presented in [8]. In case of periodic motions band-limited signals may arise, which allow us to guarantee perfect reproduction with the method proposed in this work, utilizing the minimum required number of kernels. When the band is not limited, we can guarantee the reproduction with a predefined accuracy, which is a hyperparameter that can be intuitively tuned.

II. PRELIMINARIES

A. Dynamic movement primitives for periodic motions

Dynamic movement primitives (DMP) can encode both non-periodic (discrete) and periodic (rhythmic) motions. In

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1Dimitrios Papageorgiou and Zoe Doulgeri are with the Aristotle University of Thessaloniki, Department of Electrical and Computer Engineering, Thessaloniki 54124, Greece. dimpapag@ece.auth.gr, doulgeri@ece.auth.gr
2Despina Ekaterini ARGiropoulos is with the Institute of Computer Science, Foundation for Research and Technology–Hellas, Heraklion 700 13, Greece. despinar@ics.forth.gr
this subsection the classical DMP formulation for encoding periodic motions is presented [4]. The presentation involves a motion in a 1-D space, which represents the encoding of a single coordinate of the motion.

Let \( y_d(t) \in \mathbb{R} \) be the demonstrated motion, which is periodic with known period \( T \in \mathbb{R}^+ \), i.e. \( y_d(t) = y_d(t + T) \), \( \forall t \). Notice that at least one period of this signal can be actually demonstrated, which in turn can be expanded in \( t \in (-\infty, \infty) \) due to its periodicity. The DMP for encoding \( y_d(t) \) is given by:

\[
\begin{align*}
\tau^2 \dot{y'} + \alpha (y_T - y) - \tau \dot{y} &= f(z), \\
\tau \ddot{z} &= 1,
\end{align*}
\]

(1)

(2)

were \( \tau \in \mathbb{R}^+ \) is the temporal scaling parameter, \( y \in \mathbb{R} \) the position and \( y_T \in \mathbb{R} \) the anchor point (or set point) for the oscillatory trajectory, \( \alpha, \beta \in \mathbb{R}^+ \) positive parameters of the linear part of the dynamical system and

\[
f(z) = r \sum_{i=1}^{N} w_i \Psi_i(z),
\]

(3)

with \( r \in \mathbb{R}^+ \) being the spatial scaling parameter, \( \Psi_i(z) : \mathbb{R} \to \mathbb{R} \) being the base functions (or "kernels"), \( w_i \in \mathbb{R} \) being their weights and \( N \in \mathbb{N}^+ \) their number. Last, \( z \in \mathbb{R} \) is a phase variable which is a replacement of time [4], and which evolves according to a separate dynamical system (2), the so-called “canonical system”. A typical selection for \( \Psi_i \) is the von Mises kernel given by:

\[
\Psi_i(z) = e^{h_i \cos(\frac{2\pi}{T} (z - c_i) - 1)},
\]

(4)

with \( h_i \in \mathbb{R}^+ \) being a parameter affecting its variance and \( c_i \in \mathbb{R}^+ \) its center within a single period. Notice that (4) is a periodic with respect to \( z \) Gaussian-like function, with period equal to \( T \), as shown in Fig. 1 in blue color. Furthermore, due to the linear relationship between \( z \) and \( t \), which is \( z(t) = \frac{t}{\tau} \) (solution of eq. (2) with \( z(0) = 0 \)), \( \Psi_i(z(t)) \) will be periodic also with respect to \( t \). To uniformly place \( N \) base functions, one has to define \( c_i = i \frac{T}{N} \) and \( h_i = N \).

Equation (5) is derived from (1). The encoding of the motion is done by finding the appropriate \( w_i \)'s to optimally approximate \( f_d(t) \) by \( f(z(t)) \). Hence the number of base functions \( N \) in (3) affects the accuracy of this approximation. The optimal weights \( w^* = [w_1^*, ..., w_N^*]^T \) can be found by

\[
w^* = \arg \min_w \phi,
\]

(6)

where

\[
\phi \triangleq \int_0^T e_j^2(t)dt,
\]

(7)

is the energy of reproduction error within a single period, with

\[
e_j(t) \triangleq f_d(t) - f(z(t))
\]

(8)

being the reproduction error.

The optimization problem (6), in discrete time, constitutes a least square problem which is relatively computationally heavy. The utilization of the locally weighted regression (LWR) method [9] is proposed in [4], which yields a sub-optimal but computationally lighter solution, according to which the weights are found by:

\[
w_i^* = \arg \min_{w_i} \int_0^T \Psi_i(z(t)) (f_d(t) - w_i z(t))^2 dt,
\]

(9)

which has the following analytic solution, in discrete time:

\[
w_i^* = \frac{\Psi_i^T f_d}{r \sum_{i=1}^{N} \Psi_i},
\]

(10)

with \( \Psi_i = [\Psi_i(z(T_r))...\Psi_i(z(N_r T_r))]^T, T_r \in \mathbb{R}^+ \) being the sampling period of the recorded time series and \( N_r = \frac{T}{T_r} \). One major advantage of the LWR method is that each weight \( w_i^* \) (10) is learned independently of all other weights [10].

**B. Nyquist-Shannon sampling theorem**

Consider a generic (periodic or non-periodic) signal \( f_d(t) \in \mathbb{R} \) with a frequency content limited within \( [0, F_b] \), with \( 0 < F_b < \infty \). The signal can be completely determined by the series of infinite length \( f_d(i \Delta t), i = -\infty, ..., \infty \), where \( \Delta t \leq T \), the sampling period with \( T = \frac{1}{2 F_b} \) being the Nyquist-Shannon sampling period [11]. The original signal can be perfectly reconstructed via the Whittaker-Shannon interpolation, which is given by:

\[
f_d(t) = \sum_{i=-\infty}^{\infty} f_d(i \Delta t) \Psi_i(t),
\]

(11)

with

\[
\Psi_i(t) = \text{sinc} \left( \frac{\pi}{\Delta t} (t - i \Delta t) \right),
\]

(12)

where

\[
\text{sinc}(t) \triangleq \begin{cases} \frac{\sin(t)}{t} & \text{if } t \neq 0 \\ 1 & \text{otherwise} \end{cases}
\]

(13)
C. Sampling theorem for periodic signals

Let us now consider a periodic signal \( f_d(t) \in \mathbb{R} \) with a frequency content limited within \([0, F_b] \), with \( 0 < F_b < \infty \), having a period of \( T > \frac{1}{F_b} \), i.e. \( f_d(t) = f_d(t + T) , \forall t \). The signal can be completely determined by the series of finite length \( f_d(i\Delta t) , i = 1, \ldots, N \), where \( \Delta t \leq T_s \) is the sampling period selected such that \( N \triangleq \frac{T}{\Delta t} \in \mathbb{N} \) and \( N \) the number of samples within one period. The original signal can be perfectly reconstructed via the following interpolation which is now represented by a finite summation:

\[
 f_d(t) = \sum_{i=1}^{N} f_d(i\Delta t) d\Psi_i(t),
\]

where

\[
d\Psi_i(t) = \begin{cases} D_N(t - i\Delta t) & \text{if } N \text{ is odd} \\ D_N(t - i\Delta t)\cos\left(\frac{n\pi}{\Delta t} (t - i\Delta t)\right) & \text{if } N \text{ is even} \end{cases}
\]

with \( D_N(t) \) being the Dirichlet kernel, depicted in Fig. 1, given by:

\[
 D_N(t) \triangleq \begin{cases} \sin\left(\frac{n\pi}{\Delta t} t\right) / n\pi & \text{if } t \neq \kappa T, \kappa \in \mathbb{Z} \\ (-1)^{\lceil\kappa(N-1)\rceil} & \text{if } t = \kappa T, \kappa \in \mathbb{Z} \end{cases}
\]

The perfect reconstruction achieved by (14), can be easily proven by utilizing (11), (12) given the periodicity of \( f_d \) and utilizing the following identity, which is proven in [12], [13]:

\[
 \sum_{n=-\infty}^{\infty} \sin(\pi n N \Delta t) = 0.
\]

III. Encoding a periodic motion

Inspired by the sampling theorem for periodic signals (Subsection II-C), we propose the utilization of the dynamical system presented in (1), (2) and the modulation term given by:

\[
 f(z) = r \sum_{i=1}^{N} w_i d\Psi_i(z).
\]

For the learning of the \( w_i \)-s we consider two cases, namely the case of perfectly encoding a band limited \( f_d(t) \) and the case of encoding a generic periodic \( f_d(t) \) with a predefined accuracy of reproduction. Notice that, in contrast to the discrete motions case treated in [8], the appearance of a band-limited \( f_d(t) \) is possible in real applications, due to the fact that \( f_d(t) \) is a periodic function of time.

A. Encoding band-limited motions

Consider the case of a band limited \( T \)-periodic signal \( f_d(t) \), i.e. a signal with finite number of Fourier coefficients, computed based on a demonstration \( y_d(t), \dot{y}_d(t), \ddot{y}_d(t) \) via (5). This signal could be a harmonic oscillation, or a finite linear combination of harmonic oscillations. Common examples in which such motions may occur are the shake-motion during a handshake, or the stirring of a liquid material having an elliptic or “8”-like pattern. Furthermore, let \( F_b > \frac{1}{T} \) be the maximum frequency of \( f_d(t) \).

In this case the optimal \( w_i \)-s that resolve the optimization problem (6) can be found by just sampling the demonstrated modulation term, as follows:

\[
 w_i = \frac{1}{r} f_d(i\Delta t), i = 1, \ldots, N,
\]

with \( N > [2TF_b] \) and \( \Delta t = \frac{T}{N} \). The algorithmic complexity of the learning procedure, in this case, is \( O(N) \), while the algorithmic complexity of the LS and LWR methods are \( O(N^3) \) and \( O(N \times N_s) \) respectively, with \( N_s \in \mathbb{R}^+ \) being the total number of samples of the demonstrated motion.

Remark 1: Notice that each weight \( w_i \), calculated by (20) is learned independently of all other weights, similarly to the LWR training method. Hence, when an upper bound for the frequency of the motion is known a priori, one can learn the weights on-line during the demonstration, with the algorithmic complexity in each step being \( O(1) \).

Theorem 1: The dynamical system (1), (2), (19) can perfectly encode any band limited periodic motion \( y_d(t), \dot{y}_d(t) \), with period \( T \) and upper frequency \( F_b \), i.e. \( y(t) - y_d(t) = 0 \) and \( \dot{y}(t) - \dot{y}_d(t) = 0 \) for all \( t > 0 \), by setting the \( w_i \)-s according to (20), starting from the same initial state \( y(0) = y_d(0), \dot{y}(0) = \dot{y}_d(0) \) and having the same anchor point \( y_T \), as well as the same temporal and spatial scaling parameters \( \tau, r \).

Proof: After substituting \( w_i \) from (20) to (19), utilizing (14) and given that \( f_d(t) \) is band limited by \( F_b \), the following holds:

\[
 f(t) = f_d(t), \forall t > 0.
\]

By substituting \( f_d(t) \) from (5) to (21) and employing (2) we get:

\[
 \tau^2 (\ddot{y} - \ddot{y}_d) + \alpha \tau (\dot{y} - \dot{y}_d) + \alpha \beta (y - y_d) = 0.
\]

Given that \( y(0) = y_d(0) \) and \( \dot{y}(0) = \dot{y}_d(0) \), (22) implies a perfect tracking, and thus \( y(t) = y_d(t) \) and \( \dot{y}(t) = \dot{y}_d(t) \) for all \( t > 0 \).

B. Encoding generic periodic motions with a predefined accuracy

Let us now consider the case of a generic \( T \)-periodic signal \( f_d(t) \) with bounded power, i.e. bounded energy within a single period, and equal to:

\[
 P \triangleq \int_{0}^{T} |f_d(t)|^2dt.
\]

Utilizing the Parseval’s power theorem for this signal, we get:

\[
 \sum_{k=-\infty}^{\infty} |a_k|^2 = P,
\]

with \( a_k \in \mathbb{C} \) being the Fourier coefficients. Notice that, in case of a band limited \( f_d(t) \), the summation of (24) becomes finite and thus we are able to perfectly encode the signal according to the previous subsection. However, when \( f_d(t) \)
is not band-limited, $f_d(t)$ is approximated with errors. In particular, let us define the ideally filtered $f_d(t)$, as follows:

$$\hat{f}_d(t) = \sum_{k=-N_c}^{N_c} a_k e^{jk \frac{2\pi}{T} t},$$

which is constructed after applying a Fourier-filter to $f_d(t)$, i.e. by taking into account only the $N_c \in \mathbb{N}$ first coefficients. The filtered signal $f_d(t)$ can now be perfectly reconstructed by $f(z(t))$, i.e. $f(z(t)) = \hat{f}_d(t), \forall t$, based on Theorem 1. In this case the reproduction error is calculated from (8) and by calculating the Fourier series of each term, as follows:

$$e_f(t) = f_d(t) - f(z(t)) = f_d(t) - \hat{f}_d(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk \frac{2\pi}{T} t} - \sum_{k=-N_c}^{N_c} a_k e^{jk \frac{2\pi}{T} t} = \sum_{k=-\infty}^{-N_c} a_k e^{jk \frac{2\pi}{T} t} + \sum_{k=N_c+1}^{\infty} a_k e^{jk \frac{2\pi}{T} t}.

The power of reproduction error, i.e. the energy within a period, after invoking the Parseval’s power theorem for $e_f(t)$ and given the symmetry of the magnitude of the coefficients $a_k$, is given by:

$$\phi = \sum_{k=-\infty}^{-N_c} |a_k|^2 + \sum_{k=N_c+1}^{\infty} |a_k|^2 = P - \sum_{k=-N_c}^{N_c} |a_k|^2. \quad (27)$$

Let, $h \in [0, 1]$ be a predefined, selected by the designer, accuracy reflecting the desired value of the ratio between the power of the encoding error and the power of the signal, which is defined as:

$$h \triangleq \frac{\phi}{P}. \quad (28)$$

Due to the relationship between the Fourier coefficients and the Fourier transformation in periodic signals, one can rewrite (27) employing the Fourier transformation $F_d(\omega) \in \mathbb{C}$ instead of the coefficients and utilizing (28), as follows:

$$\frac{1}{2\pi} \int_{-\frac{2\pi}{T}}^{\frac{2\pi}{T}} |F_d(\omega)|^2 d\omega = P(1-h) = \phi, \quad (29)$$

Notice that the calculation of $\phi$ via the left hand-side of (29) involves the Fourier transform, which can be calculated utilizing the FFT algorithm and therefore is computationally lighter than (27). In particular, in this case, the algorithmic complexity of the FFT, which precedes the computation of $w_i$-s, is dominant and therefore the algorithmic complexity of the proposed method is $O(N_s \log N_s)$, where $N_s \in \mathbb{R}^+$ is the total number of samples of the demonstrated motion. In order to encode the demonstrated behavior $y_d(t)$, $\hat{y}_d(t)$ with this accuracy, one has to follow the steps below:

1) Calculate $f_d(t), t \in [0, T]$ from (5).
2) Calculate the power of the signal $P$ from (23).
3) Calculate the FFT of the signal, $F_d(\omega)$.
4) Find the basic period $T$ of the signal utilizing $F_d(\omega)$, i.e., the (non-zero) coefficient with the lowest frequency, or via adaptive frequency oscillators [2]. Alternatively, if only one period is demonstrated, the basic period is equal to the duration of the demonstration.
5) Find the minimum required base functions per period $N = 2N_c$ from (29). More specifically, one has to gradually perform the integration on the left handside of (29) until reaching the quantity $P(1-h)$.
6) Produce $f_d(t), t \in [0, T]$ by filtering $f_d(t), t \in [0, T]$ with a low pass filter having a cut-off frequency of $\omega_c = \frac{2\pi}{NT}$, to avoid aliasing.
7) Uniformly sample the filtered signal, $f_d(t), t \in [0, T]$, using the $N$ samples, to get $w_i$-s from (20).

Remark 2: Notice that a continuous $f_d(t)$ in time is consider in the analysis above. However, in most of the cases the demonstrated trajectory is recorded and provided as a time series $f_{d,k} \triangleq f_d(kT_r), k = 1, \ldots, N_r$, having a relatively high sampling frequency $F_r = \frac{1}{T_r}$, with $N_r \in \mathbb{N}$ being the total number of recorded samples for which it holds $N_rT_r = T$. Given only the availability of this time series, the values at the instances dictated by (20) may not be directly available. In order to avoid this problem, one can select $N$ such that $N > 2TF_r$ and also $\frac{2\pi}{N} = \frac{2\pi}{NT} = \lambda$ with $\lambda$ being an integer. In this way, the values at the instances of (20) will always be available, as $\Delta t$ will be an integer multiple of the sampling period of the recording $T_r$, and thus $w_i = \frac{1}{T} f_d(i\lambda T_r) = \frac{1}{T} f_{d,i\lambda}$, $i = 1, \ldots, N$.

C. On-line encoding of generic periodic motions rejecting a predefined frequency content

In order to utilize the proposed model on-line, i.e. during the demonstration of one period of the motion, one has to select a priori a cut-off frequency $F_c$, which corresponds to the upper limit of the frequency content for the encoding. Some cases in which such a frequency can be predefined are the following:

1) When the demonstration is performed by a human. In this case the human arm impedance can be taken into account, which acts as a low pass filter of any intended by the human motion [14] and hence defines an upper bound on the frequency of the demonstration.
2) Safety standards or even social factors during human-robot collaboration or coexistence dictate slow motions. Hence an upper frequency limit could be estimated in this case.
3) When information about the frequency content of the noise is available, i.e. the noise frequency band.

In all the above cases, the number of basis functions can be found by utilizing the cut-off frequency $F_c$, via $N \geq 2TF_c$. After finding $N$, steps 6 and 7 should be followed, utilizing any on-line low-pass filter. Notice that, in this case, the proposed method’s algorithmic complexity in each step is determined by that of the low pass filter, which is $O(N_f)$ with $N_f \in \mathbb{R}^+$ being the number of coefficients of the discrete filter; e.g. for a second order low pass IIR filter $N_f = 2$. 

IV. SIMULATION RESULTS

To evaluate the proposed DMP model we consider the following signal for representing the demonstrated motion segment:

\[
y_d(t) = 5 \sin \left( \frac{8 \pi}{3} t \right) + 0.5 \cos \left( \frac{2 \pi}{3} t + 0.5 \right) - 3 \sin \left( \frac{14 \pi}{3} t + 15 \right),
\]

which is periodic with a period of \(T = 3\) s and band-limited with upper frequency \(F_b = \frac{7}{3}\) Hz. The predefined accuracy of reproduction is set to \(h = \frac{\phi}{F_b} = 10^{-3}\). Based on \(h\) and after calculating the FFT of \(f_d(t)\), which is depicted in Fig. 2, the minimum required number of base functions is found \(N = 15\), following the procedure described in the 5th step of the aforementioned procedure. For the anti-aliasing filter of the 6th step of the learning procedure, we utilized a zero-phase FIR low pass filter with 100 coefficients.

\[
|F_d(\omega)|\quad[10^{10}]
\]

Fig. 2: Fourier of \(f_d(t)\).

The resulted trajectories are depicted in Fig. 3, for both the Dirichlet based DMP (dDMP) and the formulation utilizing von Mises kernels trained via both the the LWR method (mDMP) and the least square method (mDMP-LS). Notice that, due to the fact that the signal is band limited with \(F_b = \frac{7}{3}\) Hz, we are able to perfectly reproduce \(f_d\) utilizing \([2TF_b]\) = 14 kernels with the proposed DMP model. In particular, the achieved accuracy ratio is \(\frac{\phi}{\phi} = 1.6478 \times 10^{-5}\), which is less than the predefined accuracy ratio \((h = \frac{\phi}{\phi} = 10^{-3})\). Notice that the von Mises based DMP trained via LWR, achieved an accuracy of \(\frac{\phi}{\phi} = 0.515\). It is worth noting that after following a trial and error procedure, we concluded that more than 700 kernels are required for achieving the predefined accuracy utilizing the mDMP with LWR. Furthermore, notice that utilizing the least square method (mDMP-LS), an accuracy ratio of \(\frac{\phi}{\phi} = 0.0692\) is achieved with 14 kernels. The widths of the Gaussian kernels are selected as suggested in [15], which accounts for the density of the kernels.

V. EXPERIMENTAL RESULTS

For the experimental evaluation two scenarios are considered, each one having a different complexity: the shake motion of a handshake, and the relatively more complex task of brush painting a symbol on a planar surface, as shown in Fig. 4.

A. Brush painting on a planar surface

In this experiment, the brush painting of a surface is considered, as shown in Fig. 4. A single cycle of execution involves a) the dipping of the brush in the pigment, which is placed on a palette, b) the approach of the surface, c) the painting of a symbol on the surface and d) the return to the palette for the next dipping. Our aim is to paint the symbol in three consecutive locations along the \(x\)-axis. The accuracy in the pattern of the motion plays a crucial role in the distortion of the painted symbol, the sufficient dipping and the interaction forces between the robot and the environment. A brush tool with a handle is attached to the manipulator’s wrist and the motion is demonstrated by the human kinesthetically, as shown in Fig. 4a. The accuracy of the path of the demonstrated motion \(y_d(t)\) is shown in Fig. 5 in black line. The period of the motion is equal to \(T = 15.25\) s and a single period is demonstrated and utilized for the training of DMP. Therefore, the basic period is found by the duration of the demonstration. During the autonomous execution, to achieve our task, the spatial scaling factor \(r\) is increased in the direction of \(x\)-axis, in each period, i.e. the values of \(r_x\) in each period are \(r_x = \{1, 1.2, 1.4\}\) respectively.

The minimum required number of kernels is calculated to be \(N_x = 20, N_y = 26, N_z = 30\) in each axis respectively, for a desired accuracy of \(h = 0.001\); this number of kernel functions is utilized for all the presented DMP approaches. The resulted painted symbols are shown in Fig. 4 and the corresponding paths and trajectories are depicted in Fig. 5 and Fig. 6 respectively. To calculate the aforementioned accuracy ratio, only the first period of the motion is taken into account and compared with the motion demonstration, for which \(r = 1\) in every axis. Notice that the proposed approach for automatically calculating the number of basis functions is not affected by the fact that only a single period is demonstrated, or by the fact that there may be a small inconsistency between the first and last values of the demonstrated motion due to human variations. Notice the significant distortion of the painted symbol with mDMP (utilizing the LWR method), which is reflected by the resulted accuracy ratios that are \(\frac{\phi}{\phi} = [9.6 \ 35.1 \ 60.6] \times 10^{-3}\).
Fig. 4: Brush painting experiment. a) Motion demonstration captured kinesthetically, b) encoding with dDMP, c) encoding with mDMP, d) encoding with mDMP-LS.

For the mDMP and $\phi_P = [0.3 \ 0.8 \ 1.3] \times 10^{-3}$ for the dDMP. However, the encoding accuracy achieved by training the mDMP model via the least square method, whose path and trajectory are depicted in Fig. 5 and Fig. 6 respectively, in red color (mDMP-LS), is similar to the proposed model, namely $\phi_P = [0.4 \ 0.5 \ 0.7] \times 10^{-3}$. However by utilizing the LS method we can not predetermine the encoding accuracy and consequently the number of base functions, which is a hyperparameter that has to be manually tuned. Furthermore, the calculation of the weights are not independent of each other, as opposed to the proposed approach.

Fig. 5: The path of the end-effector during the execution of the task.

B. Shake motion of a handshake

Handshaking is a common physical interaction utilized when humans are introduced to one another for the first time, or in greeting each other. The accuracy in the pattern of the motion plays a crucial role to the human-likeness of the interaction and may affect the acceptance of the robot by the human. In this experiment, a wooden hand is attached to the manipulator’s wrist, the motion is demonstrated by a human kinesthetically, as shown in Fig. 7, and only the z-axis (the direction of the gravity) is encoded as it constitutes the main direction of the motion during a handshake. A period of the demonstrated modulation term $f_d(t)$, calculated by (5), is shown in Fig. 8, both in the time and frequency domain. The basic frequency of the motion is found to be $\frac{1}{T} = 0.861$ Hz and a single period is utilized for the training of both DMPs (dDMP and mDMP).

Fig. 7: Kinesthetic demonstration of the shake motion.

The minimum required number of kernels are calculated to be $N = 4$ for a desired accuracy of $h = 2 \times 10^{-3}$, after following the procedure detailed in the 5th step of the learning procedure; this number of kernel functions is utilized for both DMP models. The reproduced shake motion is depicted in Fig. 9 for both DMP models for a duration of three periods. Notice the significant distortion of the shake-motion pattern occurred by mDMP, which is also reflected by the accuracy ratios which are calculated to be $\phi_P = 44.3 \times 10^{-3}$ utilizing the mDMP and $\phi_P = 1.1 \times 10^{-3}$ utilizing the proposed dDMP. Similarly to the previous experiment, in order to
find the maximum possible encoding accuracy that could have been achieved by the mDMP model, we also train the mDMP via the least square (LS) method; the results of the newly trained mDMP are also depicted in Fig. 9 in red color (mDMP-LS). Notice that the maximum possible encoding accuracy achieved by the mDMP model, in this scenario, is still not sufficient for the appropriate reproduction of the pattern, possibly due to the small number of kernel function involved in this experiment ($N = 4$).

VI. CONCLUSIONS

In this work, the utilization of Dirichlet (periodic sinc) base functions in DMPs for encoding periodic motions is proposed. The off-line and on-line training are presented and the cases of a band-limited and a generic motion are taken into account. In the case of band-limited motions, i.e., the finite linear combination of harmonic oscillations, the motion is perfectly reproduced, while in the case of a non band-limited motion, the reproduction accuracy can be predefined, leading to the minimum required number of kernels. The approach is tested through simulations and experiments, which validate the achievement of the predefined reproduction accuracy by utilizing the automatically found number of base functions.

REFERENCES


