On multi-layer game-theoretical modelling of spectrum markets and cognitive radio networks

FORTH-ICS TR 423, AUGUST 2011

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ABSTRACT

This paper presents a powerful innovative multi-layer game theoretical modelling framework for the evolution of spectrum markets. It integrates models of the channel, client and network operator (provider) paradigms, wireless infrastructures, types of interaction, and price adaptation. This modelling framework incorporates an extensive set of parameters that allow the modelling of various complex interactions of CRNs entities and business-driven cases in *multiple spatio-temporal scales* in a realistic manner. It also provides a *modular simulation* environment that implements this framework to enable researchers to instantiate various models and perform comparative assessments of spectrum-sharing and spectrum-provision mechanisms. To address several practical and systems-based issues, we proposed two novel mechanisms: a price adaptation algorithm for providers and the *u-map*, a user-centric community-based mechanism that enables clients to record and upload their feedback about their QoE during a call in a shared location-based database. To illustrate how the proposed modelling framework and simulation platform can be used, this paper analyzes the evolution of a cellular-based market that uses the proposed price adaptation algorithm. In the context of this market, we also evaluated the u-map and show how it can improve the network operator selection process. Finally, we discuss how this framework and simulation platform can be extended to analyze various spectrum markets and cognitive radio networks (CRNs).

1. INTRODUCTION

Cognitive radio networks (CRNs), an emerging disruptive technology, aims to improve spectrum utilization, enabling dynamic spectrum use. This research focuses on the design of a complete *multi-layer* modelling framework of CRNs, incorporating both systems and business aspects using statistical mechanics, game theory and economics. Its main components include the channel, network operators/providers, wireless network infrastructure and access, and clients.

A distinct characteristic in modelling the evolution of such markets is the *multiple time and spatial scales* in which the various parameters, interactions, and decision making mechanisms are manifested and/or need to be studied. For example, the rate at which a network operator changes its prices for its spectral resources is often smaller than the rate at which clients (e.g., primary or secondary devices) demand for spectral resources or select a provider. The modelling and simulation of such markets in fine-detail (i.e., microscopic-level) results in an extremely large number of events, generated by their entities and their (often complex) interactions. From a modelling and computational point of view however, it can be very expensive to keep track of all the details and not amenable to analysis. As shown in statistical mechanics, it is actually unnecessary to keep track of all details, when mesoscopic or macroscopic phenomena, which involve a very large number of clients, need to be analyzed. This observation motivated us to propose an innovative, powerful *multi-layer* modelling framework and a modular simulation environment that implements this framework to enable researchers to instantiate various models and perform comparative assessments of spectrum-sharing or spectrum-provision mechanisms in various *spatio-temporal scales*. The framework allows researchers to capture different networks, types of interactions, negotiation strategies, price-setting mechanisms, provider selection approaches, information availability and trust among entities at the appropriate level of detail.

Most of the proposed models for CRNs focus on a specific sub-problem/aspect of CRNs omitting their inherent features. Specifically, they can be classified into two categories, namely, the *microscopic*- and the *macroscopic*-level ones. The microscopic-level approaches consider interactions among secondary devices at a very fine spatial level, mostly assuming a limited number of primary devices due to the high computational complexity (e.g., [1, 4, 12, 17, 5, 19, 18]). On the other hand, the macroscopic approaches focus on the revenue of providers, considering only an "average" (over large temporal or spatial scales) behaviour of secondary devices (e.g., [8, 7, 10, 11]). Unlike these approaches, this framework models the interactions of the entities at *several spatial scales*, from large metropolitan areas to small neighborhoods (e.g., within the coverage of a wireless access point), enabling the instantiation of various parameters at *different time granularities*. Moreover, it provides the set of models and mathematical transformations that allow to "scale up or down" a modelling environment in order to analyze a certain phenomenon in mesoscopic or microscopic level. At the microscopic level of the framework, the various entities are modeled in fine temporal and spatial detail. On the other hand, the mesoscopic level exhibits various *aggregations*. In a "coarse-graining" procedure that results to a *loss of information in a controlled and hierarchical fashion* [9], the individual entities (e.g., clients) are replaced by local averages over space and/or time. For example, clients are modeled as a *population* with certain attributes, computed as *spatial averages* of the characteristics of the individual users of that population. To the best of our knowledge, it is the *only* modelling framework that attempts to incorporate such an extensive set of parameters that allow the modelling of various complex interactions of CRNs entities and business-driven cases in *multiple spatio-temporal scales* in a realistic manner.

Providers and clients (i.e., agents that sell/sublease or buy spectral resources, respectively) are modeled in a game-theoretical framework. Updating mechanisms are chosen to model their learning and decision mechanisms and can be instantiated by a rich set of strategy updating rules (e.g., best-response, perturbed best-response, mutation, imitation). The choice of utilities, spatial structure and (stochastic) updating mechanisms define completely a multi-agent evolutionary game.

Two important aspects of this framework are the amount of information available to agents and the type of their interactions. This paper makes several contributions both in systems and modelling; First, it presents this innovative multi-layer game-theoretical framework and discusses the theoretical issues and properties (in Section 2). Then, it describes a simulation platform that implements this framework and can instantiate different spectrum and cognitive radio markets (in Section 3). As spectrum and cognitive radio markets evolve, various mechanisms that improve the wireless access have been proposed. We envision a novel system that enables clients to upload their feedback/measurements about providers and their quality-of-experience (QoE) of their access in a shared map-based database (superimposing their reports on the region of the map that corresponds to their location). Based on this feedback and statistics on these measurements, in a grass-root, user-centric fashion, a QoE/coverage map can be built. We call this system *u-map*. Clients may use statistics provided in this map to select a network operator. Network operators could potentially employ such statistics to improve their network (capacity planning). Furthermore, they can use the collected statistics (e.g., about user characteristics, requirements and populations), to instantiate the market in the modelling framework/simulation platform and analyze its evolution. To illustrate how the proposed framework and simulation platform can be used, we analyzed the evolution of the most typical spectrum provision paradigm, a cellular-based market (in Section 3). In the context of this market, addressing practical and systems-based issues, we proposed different access paradigms (e.g., subscribers and rechargeable card users) and a novel price adaptation algorithm. Section 4 analyzes the evolution of this market under different client populations and information availability (e.g., u-map) scenarios. Finally, Section 5 summarizes our main findings and future work plans.

2. MODELLING FRAMEWORK

The main entities of the modelling framework are the clients, providers, and channels. Each provider owns a spectrum band that consists of a number of discreet channels and offers them for an appropriate remuneration. A client requests for a channel from a provider.

Consider a set of clients U and a set of providers P. Each provider (e.g., $p \in P$) maintains an infrastructure of base stations BSs (set B_p). The set of channels (of all providers) in the spectrum is indicated by Y, while the set of available transmission power levels is T. Denote as B the set of all BSs (of all providers) ($B = \cup_{p \in P} B_p$), and $BY_p = \{(x, y) \in B_p \times Y :$ channel y is allocated to BS $x\}$ the set of BS and channel pairs that belong to the provider $p \in P$. Assume that $BY = \bigcup_{p \in P} BY_p$.

Each client u is characterized by two constraints, namely, the *price tolerance threshold* $C_{max}(u)$, i.e., the maximal price that the client u accepts to pay, and the *target transmission rate* ($R_{tar}(u)$) i.e., the minimal transmission rate that the client u aims to achieve. When these two constraints cannot be satisfied by any provider, the client chooses to remain disconnected. BSs and clients have a physical location in a region $A \subset R^2$. The function $L : U \cup B \to A \subset R^2$ maps each client and each BS to their corresponding locations.

The set of strategies of a client u is $S_u = BY \times T \cup \{e\}$. We represent the strategy of a client $u \in U$ as a triplet $\eta(u) = (\eta_b(u), \eta_c(u), \eta_\tau(u))$, where $(\eta_b(u), \eta_c(u)) \in BY$ are the BS and channel, respectively, to which the client u connects, while $\eta_\tau(u)$ is the value of transmission

Table 1: **Main modelling parameters**

Notation	Description
A	region with client/BS distribution
L	mapping of client/BS to location in A
U	set of clients
B	set of BSs
P	set of providers
Y	set of channels
T	set of trx power levels
Providers	
$\sigma(p)$	strategy of provider p
B_p	BSs belonging to provider $p \in P$
BY_p	$\{(x, y) \in B_p \times Y :$ channel y is allocated to BS $x\}$
BY	$\cup_{p \in P} BY_p$
S_p	set of strategies
$F(p; \sigma, \eta)$	utility function of provider p
Clients	
$\eta(u)$	strategy of client u
$\eta_b(u)$	BS to which u connects
$\eta_\tau(u)$	transmission power of u
$\eta_c(u)$	channel to which u connects
$I(u; \eta)$	observed interference when u selects $\eta(u)$
$R(u; \eta)$	achievable transmission rate
$C(u; \eta, \sigma)$	financial cost of selecting $\eta(u)$
$C_{max}(u)$	price tolerance of u
$R_{tar}(u)$	target transmission rate of u
S_u	set of strategies of u
$F(u; \sigma, \eta)$	utility function of client u

power that this client invests. Moreover, the strategy e represented by the triplet $(0, 0, 0)$ indicates the disconnection state. The set of strategies of a provider p is $S_p = \{s_{p,1}, s_{p,2} \ldots, s_{p,M_p}\}$, where $s_{p,i}$ is the $i-$th choice of price of the provider p.

The *state* of the system [1] corresponds to the positions of all clients and BSs (given by the function L) and the strategies of all providers and clients $\sigma = (\sigma(p))_{p \in P}$, where $\sigma(p) \in S_p$ for each $p \in P$, and $\eta = (\eta(u))_{u \in U}$, where $\eta(u) \in S_u$ for each $u \in U$.

Each provider seeks to optimize its revenue by increasing both market share and price. The utility function $F(p; \sigma, \eta)$ for a provider $p \in P$, given strategies σ and η for providers and clients, respectively, is given by

$$F(p; \sigma, \eta) = \sum_{u \in U} \sum_{(x,y) \in BY_p} \delta(\eta_b(u) - x, \eta_c(u) - y)\eta_\tau(u)\sigma(p) + \sum_{u \in U} \sum_{(x,y) \notin BY_p} \delta(\eta_b(u) - x, \eta_c(u) - y)(C_{max} - \eta_\tau(u)\sigma(p))$$

where C_{max} is the average maximum price that clients can tolerate, i.e., $C_{max} = |U|^{-1} \sum_{u \in U} C_{max}(u)$, and $\delta(,)$ is a function (Kronecker's delta) that returns 1 when both arguments are zeros, otherwise the value 0. The first term in the utility $F(p; \sigma, \eta)$ expresses the objective of a provider to charge the maximal price to its customers, while the second term expresses the objective to increase the number of its clients. To penalize an aggressive increase of the transmission power, the providers adopt a pricing scheme that charges the clients proportionally to the transmission power they invest.

Each client $u \in U$ has two competing objectives, namely, to maximize the achievable transmission rate $R(u; \eta)$, and to minimize the financial cost $C(u; \eta, \sigma)$ to achieve this transmission rate. The transmission rate $R(u; \eta)$ is computed based on the Shannon capacity theorem, although more sophisticated models that take into consideration the modulation schemes can also be incorporated easily [13]:

$$R(u; \eta) = \mathcal{B} \log_2 \left(1 + \frac{\eta_\tau(u) G(L(u), L(\eta_b(u)))}{I(u; \eta)}\right),$$

where $\mathcal{B}$ is the channel width and $G(x, x')$ is the channel gain between a transmitter and a receiver at positions x and x', respectively. The channel gain has the form $G(x, x') = PL(d_0)(\|x - x'\|_2/d_0)^{-n} X$, with a path loss exponent n, the channel gain $PL(d_0)$ at the reference distance d_0, and the shadowing parameter X, a log-normal random variable of zero mean. The amount of interference power that the client u observes when it connects to the BS $\eta_b(u)$ at the channel $\eta_c(u)$ is given by the following formula:

$$I(u; \eta) = \sum_{\{u' : \eta_c(u) = \eta_c(u')\}} \eta_\tau(u') G(L(u'), L(\eta_b(u))) \quad (1)$$

The financial cost is given by $C(u; \eta, \sigma) = \eta_\tau(u)\sigma(p(\eta_b(u)))$, where the mapping $p(\eta_b(u))$ returns the provider to which the BS $\eta_b(u)$ belongs. Note that the utility function can be extended to incorporate more user preference aspects. For example, if QoE evaluations are available (e.g., via the u-map, as described earlier), the objective of a client can be described as a weighted[2] sum of the transmission rate, the financial cost, and the QoE indicator, as follows:

$$Z(u; \eta, \sigma) = a_1 R(u; \eta) - a_2 C(u; \eta, \sigma) + a_3 q(\eta(u)) \quad (2)$$

To complete the definition of the payoff function of a client, we must also take into consideration the aforementioned client constraints. The utility function of a

[1] Note that the "$;\sigma, \eta$" (or "$;\sigma, \mathbf{n}$") indicates the state of the system.

[2] The role of the weights is to perform a prioritization and "unit" transformations among the various objectives.

client $u \in U$, $F(u;\sigma,\eta)$ can now be defined as follows:

$$F(u;\sigma,\eta) = \begin{cases} Z(u;\eta,\sigma) & \text{if } \eta(u) \neq e, \\ & R(u;\eta) \geq R_{tar}(u), \\ & C(u;\eta,\sigma) \leq C_{max}(u), \\ l - \varepsilon_1 & \text{if } \eta(u) = e \\ -\varepsilon_2 & \text{otherwise} \end{cases}$$

where $l = \inf_{u,\eta,\sigma} Z(u;\eta,\sigma)$ and ε_1, ε_2 are positive numbers with $-\varepsilon_2 \ll l - \varepsilon_1$. Note that this utility function assumes that the client aims to remain always connected as long as its criteria (on price and target rate thresholds) are satisfied. Only if any of the thresholds are not satisfied, the client chooses to remain disconnected ($-\varepsilon_2 \ll l - \varepsilon_1$). Observe that in a more general case, a client may prefer to stay disconnected, e.g., under the absence of traffic demand or limited battery. The utility function can be easily generalized to consider such cases by introducing additional constraints.

Our models are agent-based, in the sense that they employ detailed properties and preferences of all clients and providers in an individual basis. Throughout the paper, the word "agent" is used to indicate a client or a provider. This approach makes very easy the integration and enrichment of these models with, as many as desired, additional individual client or provider characteristics by augmenting their strategy set with new variables and defining new corresponding utility functions.

Dynamical updating

Once the utilities of the agents have been specified, we have defined a *multi-agent game* in the sense of traditional game theory [16]. We will follow further the evolutionary game theoretic approach [14] and complement the game by specifying dynamical (stochastic) rules with which providers and clients update their strategies. The updating mechanisms are chosen to model the learning and decisions mechanisms of agents. In the context of evolutionary games, one assumes that the updating rates are based essentially only on the utility functions. In particular, in traditional game theory, one often assumes that *every agent knows the strategies of all the others* to compute its own utility function. This is a *strong assumption* which is very useful in modelling but unrealistic in practice. We will further discuss and relax this assumption later on.

(a) Updating times The updating can occur at discrete times intervals δt which, in general, may depend on the type of agents. We consider here instead continuous time dynamics: each agent is equipped with its own "clock", i.e. an exponential random variable with a rate which determines the time at which it will update its strategy. The memoryless property of the exponential random variable ensures that this gives rise to a continuous time Markov process.

(b) Updating rates A change in the strategies of providers is indicated by the transition $\sigma \to \sigma' \equiv \sigma^{p,\omega}$, where $\sigma^{p,\omega}$ is the same as σ with the difference that provider p has changed its strategy from $\sigma(p)$ to ω.

$$\sigma^{p,\omega}(p') = \begin{cases} \sigma(p') & \text{if } p' \neq p \\ \omega \in S_p & \text{if } p' = p \end{cases}$$

One can further restrict the range of the values that ω can take in each update, for instance, in a neighborhood of the current value $\sigma(p)$, in order to model an aversion to abrupt changes in the pricing. Similarly, one can incorporate more elaborate schedules in the selection of the new price ω, depending on the modelling needs.

A change in the strategies of clients is indicated by the $\eta \to \eta' \equiv \eta^{u,J}$, where $\eta^{u,J}$ is the same as η with the difference that the client u has switched from the strategy $\eta(u)$ to J.

$$\eta^{u,J}(u) = \begin{cases} \eta(u') & \text{if } u' \neq u \\ J = (J_b, J_c, J_\tau) \in S_u & \text{if } u' = u \end{cases}$$

In the context of cognitive radios, it is natural to restrict the values of the coordinates of BS J_b of the new configuration $J = (J_b, J_c, J_\tau)$ to be in the neighborhood of the current location of a client (e.g., $L(u)$). One specifies updating rates $c_p(\sigma, \sigma^{p,\omega}; \eta)$ for providers and $c_u(\eta, \eta^{u,J}; \sigma)$ for clients. In most examples considered here, the transition rates between two states depend on the payoff of both states through an appropriate function (e.g., g_p for providers and g_u for clients)

$$c_p(\sigma, \sigma^{p,\omega}; \eta) = g_p(F(p;\sigma,\eta), F(p;\sigma^{p,\omega},\eta)) \quad (3)$$

$$c_u(\eta, \eta^{u,J}; \sigma) = g_u(F(u;\sigma,\eta), F(u;\sigma,\eta^{u,J})), \quad (4)$$

where the functional forms g_p and g_u (e.g., Gibbs sampler, Metropolis) correspond to the strategy updating rules that are defined below. The generator of the continuous-time Markov process has then the form

$$\mathcal{L}f(\sigma,\eta) = \lambda_{\rm pr}\mathcal{L}_{\rm pr}f(\sigma,\eta) + \lambda_{\rm cl}\mathcal{L}_{\rm cl}f(\sigma,\eta)$$

where the rates $\lambda_{\rm pr}$ and $\lambda_{\rm cl}$ specify the time scales at which clients and providers update their strategies and

$$\mathcal{L}_{\rm pr}f(\sigma,\eta) = \sum_{p\in P}\sum_{\omega\in S_p} c_p(\sigma,\sigma^{p,\omega};\eta)\left[F(p;\sigma^{p,\omega},\eta) - F(p;\sigma,\eta)\right]$$

$$\mathcal{L}_{\rm cl}f(\sigma,\eta) = \sum_{u\in U}\sum_{J\in S_u} c_u(\eta,\eta^{u,J};\sigma)\left[F(u;\sigma,\eta^{u,J}) - F(u;\sigma,\eta)\right]$$

This can also be easily generalized for other types of updating mechanisms. [3]

[3]A generator is a compact way to write the Kinetic Monte Carlo (KMC) algorithm that implements the Markov process based on an efficient calculation of the waiting times between updatings and the transition probabilities at the updatings.

(c) Updating rules We consider here a number of useful rules, but other choices are possible, see e.g. [15].

1. Best-response dynamics This dynamics is the natural choice to model *rational* agents: each agent evaluates the utility functions for all the accessible strategies and chooses the one with the highest possible payoff. If K strategies are accessible with payoffs $F_1, \cdots F_K$ then upon updating it chooses the strategy argmaxF_i with a suitable tie-breaking rule. For example, in our model, a client would search for all available BSs and channels in its neighborhood, and all possible transmission power levels, and choose the more advantageous one (including the disconnection). This may lead to nontrivial optimization problems.

2. Generalized Gibbs sampler This dynamics models an agent who tends to act in a rational manner but makes "mistakes" which are encoded in a parameter $\delta \geq 0$. If an agent has K strategies with payoffs $F_1, \cdots F_K$, then upon updating, it will choose strategy j with probability $e^{F_j/\delta}/\sum_{i=1}^{K} e^{F_i/\delta}$. Note that as δ tends to 0 the Gibbs sampler dynamics rule tends to the best response dynamics.

3. Generalized Metropolis dynamics The Metropolis rule is well adapted to model rational agents which have a very large number of strategies to choose from. Rather than evaluating all the possible strategies, the agent picks one strategy at random, say strategy j, and compares it to its current strategy, say strategy i. Updating to strategy j occurs with probability max$\{1, e^{(F_j-F_i)/\delta}\}$, that is the agent always updates to better strategies but makes mistakes with an exponentially small probability. As δ tends to 0, the Metropolis dynamics tends to a dynamics where agents pick a new strategy, and then update it, if it corresponds to a higher payoff.

4. Imitation dynamics The agent picks one of its peers and imitates it, if the peer strategy is more favorable than its own. In the context of spatial games, one specifies a probability to choose any given one of its peer (depending for example of the distance between agents), and the agent updates with a rate $\max\{0, F_j - F_i\}$, if its current strategy is i and its peer strategy is j.

5. Mutation dynamics Often it is natural to add a certain "drift" (in the biological sense) to the strategies of agents. In that case, the agent if it has currently strategy i, upon updating, keeps its current strategy with probability $1-\mu$ and "drifts" to a randomly chosen strategy with probability μ.

The different updating mechanisms can be easily combined in a flexible way: clients and providers may have different updating rules from each other; a given agent may update by combining various behaviors, e.g. imitation and mutation, or imitation and best response; or different groups of clients can have different updating mechanism.

Incomplete information

The previous dynamics consider a well-defined model of the payoff of each provider, which is used to perform the price adaptation. As discussed earlier, this is a strong and unrealistic assumption, which we will relax. Specifically, we propose a novel price determination/adaptation algorithm which assumes that providers know *only* their own prices and the prices of their competitors and measure their own revenue. That is, we assume that providers do *not* a priori know the utility that corresponds to a price they may offer in the market. Let us introduce now this new dynamics:

6. Polynomial-based approximation of payoff A provider performs a novel price adaptation algorithm based on a *second-degree concave polynomial* approximation of the payoff function and estimates its parameters based on its own history of the game evolution. This approximation is simple yet appropriate to capture the mathematical properties of the payoff function of a provider. Specifically, each provider keeps track of the last combinations of prices that have been offered as well as the corresponding values of its revenue. It periodically fits the polynomial to the recently collected data by solving a least-square problem with the additional constraint that the polynomial is concave, formulated as a *semi-definite program.* The price is adapted by "moving towards" the direction of the partial derivative of the polynomial that corresponds to that specific provider and with a certain step. Providers adapt their prices at time instances generated via a stochastic process (for example Poisson distribution). Furthermore, notice that in the previous models (e.g., Gibbs sampler, metropolis), agents (e.g., clients) choose strategies at times generated by a Poisson process. In contrast, in this dynamics, agents may consider different distributions. In principle, to include such generalized distributions in our modelling framework is possible (e.g., general waiting times have been used in other queueing/network models) but we shall not pursue this further in the modelling part of this paper. This dynamics has been fully implemented and evaluated in our simulations (presented in the next sections). It also natural to allow client mobility. This can be implemented fairly easily in our theoretical model and has been implemented in the simulations.

Convergence and stability

There are few if any general results on the convergence of the stochastic algorithms at the level of generality used here. Let us assume that the number of clients and providers is finite so that the dynamics is given by a finite state Markov chain. Furthermore, if agents update strategies in such a way that all of the states of the Markov chain can be reached by a sequence of updating of the various agents (irreducibility), then the

Markov chain converges to a unique steady state from any initial condition (ergodicity of the Markov process). A Markov chain with perturbed best response (generalized Metropolis and Gibbs sampler) is naturally irreducible, if well-designed. The fact that unfavorable strategies are chosen (with exponentially small probabilities) makes it usually fairly easy to verify the irreducibility of the Markov chain. If the dynamics contains exclusively imitative mechanisms, then the dynamics has the intrinsic property that no new currently unused strategy is introduced in the system. For example, imagine that clients (quite unrealistically) update their strategies only by imitating their neighbors, then a newly entered provider would never gain any new clients. In this case, one does not expect convergence to a unique steady state, but rather the existence of several "absorbing" states. For example, in our model, these absorbing states could correspond to the "extinction" of a provider. However any level of rational behavior, or mutation of clients will ensure a stable algorithm. If the updating is based uniquely on best-response mechanisms, then intuitively, the Markov chain has very little stochasticity (most updates occur with probability one). The question of convergence for these systems can be extremely difficult to study, especially if its number of states is very large. One should remark further that in the limit where the number of agents tend to infinity, a number of interesting phenomena can occur. One may observe various phase transition, for example one may loose stability and uniqueness of the steady state for perturbed best-response rules, or in the contrary for imitative dynamics one may regain some form of ergodicity and observe complex steady states, see. e.g. [2].

Coarse-graining and population games

The choice of utilities, spatial structure and (stochastic) updating mechanisms define completely a multi-agent evolutionary game. This is an extremely powerful tool to model the actions of agents in as much detail as wished. However, as mentioned earlier, from a modelling and computational point of view, it can be very expensive, if at all possible, to keep track of all the details about strategies of every single agent, and actually unnecessary, if one is interested in macroscopic or mesoscopic quantities which involve averages over many clients. We shall describe here a *reduced model* where we keep full track of the strategies of the typically very few providers but "coarse-grain" the clients, constructing an *evolutionary population game* model [14] which describes the time evolution of local *populations* rather than *individual* clients. This coarse-graining procedure corresponds to *a loss of information in a controlled (hierarchical) fashion*, [9], replacing individual agents/quantities by local averages over space.

Table 2: **Coarse-grained model parameters**

Notation	Description
D(k)	Region k
n(k)	Number of clients in D(k)
$\mathcal{U}(k, b, i)$	representative client in D(k) at BS b with trx power τ_i
n(k, b, i)	Number of clients in D(k) connected at BS b with trx power τ_i
$\mathbf{n}$	vector describing client populations
$\mathcal{U}(k)$	representative client in region $D(k)$
$\overline{G}(k, b)$	avg channel gain btwn $\mathcal{U}(k)$ & BS b
$\overline{R}(k, b, i; \mathbf{n})$	avg achievable trx rate of $\mathcal{U}(k, b, i)$
$\overline{I}(k, b; \mathbf{n})$	avg interference experienced by $\mathcal{U}(k, b,)$ connected at BS b
$\overline{F}(k, b, i; \sigma, \mathbf{n})$	utility of $\mathcal{U}(k, b, i)$
$\overline{F}(p; \sigma, \mathbf{n})$	utility of provider p

We make the following assumptions: **(a)** The population in the region A is naturally divided in subgroups located in the regions $D(k)$ with $k = 1, \cdots, K$. For example, one might imagine that populations are naturally divided in neighborhoods with different densities or transmission power. We have $A = \cup_{k=1}^{K} D(k)$ and denote by $n(k)$ the total number of clients in region $D(k)$. For the sequel, we assume that $n(k)$ is fixed in time, although this could be easily modified. We only keep track of the region of a client (e.g., $D(k)$) and not its specific position ($L(u)$).

(b) Rather than keeping track of the transmission power of each client, we divide the transmission power range T into two subsets: the low and high transmission power ranges ($T = T_{low} \cup T_{high}$), and assign two representative transmission power levels τ_{low} and τ_{high}, respectively. We assign to each client a label $i \in \{0, low, high\}$, where "0" indicates the "no transmission". Of course, this could be easily generalized to more than three levels, if desired.

(c) We stop keeping track of channels choice of clients, and simply, record the BS b to which clients are connected. We define new dynamical variables $n(k, b, i)$ to indicate the number of customers in region $D(k)$ connected to BS b and investing a transmission power τ_i, i.e.,

$$n(k, b, i) = \sum_{\substack{u \,:\, L(u) \in D(k), \\ \eta_T(u) \in T_i}} \delta(\eta_b(u) - b) \tag{5}$$

for $b \in B$ and $k = 1, ..., K,\ i \in \{0, low, high\}$. We also set $n(k, 0, 0)$ to denote the number of disconnected clients in the region $D(k)$.

(d) In order to estimate the interference, recall that each provider $p \in P$ is assigned a specific frequency band that consists of a number of discrete channels and that frequency bands of distinct providers may be overlapping. Furthermore each provider allocates channels to BSs according to a frequency reuse scheme. The set of channels that are allocated to a specific BS $b \in B$ according to this scheme, is denoted as Y_b. The number of channels $|Y_b \cap Y_{b'}|$, which are shared by two BSs will be instrumental to determine the interference.

(e) We replace the channel gain by a suitable average:

$$\overline{G}(k,b) \;=\; \frac{1}{n(k)} \sum_{u:L(u)\in D(k)} G(L(u),L(b))\,.$$

Coarse-grained payoff By performing various averages we now express the payoff exclusively in terms of the vector $\mathbf{n}$ describing the various *client populations* $\mathbf{n} = (n(k,b,i))$, with $k \in \{1,\cdots,K\}, b \in B, i \in \{0, low, high\}$. We approximate the utility $F(u;\sigma,\eta)$ by a new function $\overline{F}(k,b,i;\sigma,\mathbf{n})$ which is the utility of a *representative* client in region $D(k)$ that connects to BS b and invests a transmission power τ_i. Assume that $\overline{R}_{tar}(k)$ and $\overline{C}_{max}(k)$ are the average target transmission rate and the average price tolerance threshold of the population of clients at region $D(k)$, respectively. First, we set the achievable transmission rate of a representative client in the region $D(k)$ that is connected to BS b using transmission power τ_i to be $\overline{R}(k,b,i;\mathbf{n}) = \mathcal{B}\log_2\left(1+\frac{\tau_i\overline{G}(k,b)}{\overline{I}(k,b;\mathbf{n})}\right)$ where

$$\overline{I}(k,b;\mathbf{n}) = \sum_{b'\neq b}\sum_{k'}\sum_{j} \tau_j \overline{G}(k',b)\frac{n(k',b',j)}{|Y_{b'}|}\frac{|Y_{b'}\cap Y_b|}{|Y_b|} \tag{6}$$

The interference $\overline{I}(k,b;\mathbf{n})$ (defined in the above Eq.) experienced by a representative client in the region $D(k)$ connected with BS b is derived as follows: As part of the coarse-graining, one assumes that a client selects a channel uniformly at random (from the set of channels of a BS), when it connects to that BS. Let assume a client of type j in $D(k')$, connected at BS b'. It connects to an arbitrary channel of BS b' with probability $\frac{n(k',b',j)}{|Y_{b'}|}$. Then $\tau_j\overline{G}(k',b))$ is the interference that it causes to a representative client in $D(k)$ which is connected to the same channel at the BS b. The representative client in $D(k)$ picks the same channel when it connects to BS b as the first client with probability $\frac{|Y_{b'}\cap Y_b|}{|Y_b|}$.

The utility function of a representative client in region $D(k)$ connected at BS b, with transmission level τ_i is as follows:

$$\overline{F}(k,b,i;\sigma,\mathbf{n}) = \begin{cases} \overline{Z}(k,b,i;\sigma,\mathbf{n}) & \text{if } b\neq 0, \\ & \overline{R}(k,b,i;\mathbf{n}) \geq \overline{R}_{tar}(k), \\ & \overline{C}(b,i;\sigma) \leq \overline{C}_{max}(k) \\ \bar{l}-\varepsilon_1 & \text{if } b=0 \\ -\varepsilon_2 & \text{otherwise} \end{cases}$$

where $\bar{l} = \inf_{k,b,i;\sigma,\mathbf{n}} \overline{Z}(k,b,i;\sigma,\mathbf{n})$ and

$$\overline{Z}(k,b,i;\sigma,\mathbf{n}) = a_1\overline{R}(k,b,i;\mathbf{n}) - a_2\overline{C}(b,i;\sigma) + a_3\overline{q}(k,b,i).$$

The cost that the representative client pays when connected to BS b using transmission power τ_i is $\overline{C}(b,i;\sigma) = \tau_i\sigma(p(b))$ and finally the utility function for a provider p considering the client populations is defined as following

$$\begin{aligned}\overline{F}(p;\sigma,\mathbf{n}) \;=\; & \sum_{b\in B_p}\sum_{k}\sum_{i} n(k,b,i)\tau_i\sigma(p) \\ & + \sum_{b'\notin B_p}\sum_{k}\sum_{i} n(k,b',i)(C_{max}-\tau_i\sigma(p))\end{aligned} \tag{7}$$

Coarse-grained dynamics

We can easily derive a coarse-grained dynamics for the variables σ and $\mathbf{n}$. As in the microscopic case, the definition of dynamics entails two steps: defining an updating mechanism and a corresponding rate. An updating occurs at the transition of a client in some region $D(k)$ from one BS b' and type i' to another BS b'' and type i'' with $(b'',i'') \neq (b',i')$ (recall that $b=0$ means disconnected). Therefore, the transitions are

$$\mathbf{n} \mapsto \mathbf{n}' = \mathbf{n}^{l,(b'i')(b''i'')} \tag{8}$$

where $j \in \{1,\cdots,K\}$, $b', b'' \in B$, $i', i'' \in \{0, low, high\}$, and

$$\mathbf{n}^{j,(b'i')(b''i'')}(k,b,i) := \begin{cases} n(k,b,i) & \text{if } j\neq k \\ n(k,b,i)-1 & \text{if } j=k, b=b', i=i' \\ n(k,b,i)+1 & \text{if } j=k, b=b'', i=i'' \end{cases}$$

Since the utilities depend only on the variables σ and $\mathbf{n}$, then the updating rates can also be expressed only in terms of the variables σ and $\mathbf{n}$ as in (3) and (4), based on the general strategy updating rules, such as best response, Gibbs sampler, and the coarse-grained payoffs.

The convergence and stability properties of the coarse-grained stochastic processes are closely related to the ones of the original agent-based. Under stochastic rules such as Metropolis and Gibbs sampler one obtains a unique stable steady state and the system converges to the steady state for any initial condition. The same caveat about imitation and best-response dynamics apply here too.

It is important to observe that the coarse-graining and population games methodology does not simply perform an averaging, reducing the initial client population size. The population size remains the same. Instead it provides the set of mathematical transformations and models for creating various populations, "scaling up" certain parameters to reduce the unnecessary information.

3. SIMULATION PLATFORM

The modelling framework is parameterized based on the channel, infrastructure and network topology, type of users (e.g., requirements, demand, mobility, profile), providers (e.g., access protocol, price adaptation mechanism), and their interactions, client/provider distributions, mobility, and available information. The simulation environment based on this framework is modular, in that, it can instantiate and implement different models for these parameters. For example, the channel can be modeled using large-scale propagation models (e.g., path-loss and shadowing) and small-scale models (e.g., multi-path fading).

Providers This work considers the cellular topologies of two providers, owners of spectral resources, that offer wireless access via their BSs to clients in a small city. Furthermore, we assume that the providers divide their channels into time-frequency slots according to a TDMA scheme.

Channel To simulate the channel quality, we employed the *Okumura Hata* path-loss model for small cities. Moreover, the contribution of shadowing (expressed in dB) to the channel gain at the positions of BSs follows a multivariate Gaussian distribution with mean $\mathbf{0}$ and covariance matrix defined in Eq. (9).

$$C(i,j) = \begin{cases} \sigma_s^2 & \text{if } i = j, \\ \sigma_s^2 e^{-\|L(i)-L(j)\|/X_c} & \text{if } i \neq j, \end{cases} \tag{9}$$

where σ_s is the standard deviation of shadowing (2.5 dB in our simulations), X_c is the correlation distance within which the shadowing effects are correlated [6], and $L(i), L(j)$ are the positions of the BSs i and j, respectively.

To model the effect of angular correlation of shadowing, we "represent" each BS using six points, located on a circle with center the BS position. When a client communicates with a specific BS, the contribution of shadowing to the channel gain is equal to the value that corresponds to the point representing the BS, whose direction is the closest to the direction of arrival of the signal [3].

The interference power at a BS during a time frequency slot is computed considering the contribution of all interfering devices at cochannel BSs. Moreover, cochannel BSs of the same provider may not be synchronized, resulting in "overlapping" time frequency slots, and thus, in devices that cause interference during more than one slots.

Client populations Two types of client populations are present: the *card users* and the *subscribers*. A card user selects a BS at the start of each call (dynamically), while *subscribers* choose their provider upon their arrival in the region and connect only to BSs of that provider for the remaining duration of the experiment.

A client is characterized by a *price-tolerance* threshold and a *target transmission rate* threshold (as described in Section 2). Based on their preference, clients can also be distinguished in two categories, namely the *price-preference* and *transmission rate-preference* ones. In rate preference, clients aim to optimize *only* their transmission rate when selecting a BS, given that this BS can satisfy both the price tolerance and target transmission rate thresholds (e.g., $a_1 = 1$, $a_2 = 0$, $a_3 = 0$ in Section 2, Eq.2). Clients with price preference aim to minimize the cost of acquiring a time-frequency slot, when selecting a BS, given that this BS can satisfy both the price-tolerance and target-transmission rate thresholds.

Client demand Clients generate requests to connect to a BS (i.e., *calls*). The duration of calls and disconnection periods are given by appropriate stochastic processes.

Client mobility Clients move with pedestrian speed. The position of a client at the end of a disconnection period is chosen randomly from a circular region with center the position of the client at the beginning of the disconnection and radius the maximum possible travelled distance during this period. If a client ends up in a position outside the borders of the city it is reflected back in the city. The simulation platform can be extended to consider other mobility models (e.g., vehicular mobility) and mobility traces.

User-centric QoE map (u-map) The u-map is a data structure that corresponds to a grid-based representation of a region. Each cell of the grid stores statistics about the providers and the QoE of calls. In the context of this analysis, we assume that the cell size corresponds to the simulated small city. At the end of a call, a card client reports the number of available time-frequency slots of the closest BS of each provider. Subscribers also report the number of available time-frequency slots of the closest BS of their providers. This information is uploaded and stored in a centralized database (u-map). Based on this information, the average *spectrum availability* i.e., number of available time-frequency slots of a BS of a provider, averaged over all collected measurements, is computed. Subscribers select the provider with the highest average spectrum availability. In this paper, the main purpose of the u-map is to reduce the call blocking probability. In a more general context, different type (e.g., QoE-based) of measurements can be recorded on the u-map, the cell size of the u-map may vary, and different BS/provider selection mechanisms can be employed to improve the QoE of a user.

4. PERFORMANCE EVALUATION

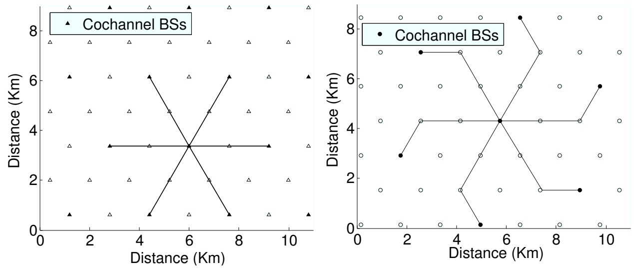
4.1 Simulation scenarios

Wireless network infrastructure The simulation platform considers a small city, represented by a grid of 11 Km x 9 Km. Each provider has a cellular network that consists of 49 BSs placed on the sites of a triangular grid, with a distance between two neighboring sites of 1.6 Km. Moreover, each provider owns bandwidth of 5.6 MHz, that is divided into 28 channels of 0.2 MHz width. These channels are allocated to BSs according to a frequency reuse scheme with spatial reuse factors of 4 and 7, for Provider 1 and Provider 2, respectively. The closest BSs at the same frequency band as a given BS in a topology with a spatial reuse factor of 4 can be located by "moving" two steps towards any direction on the grid. On the other hand, in a topology with a spatial reuse factor of 7, by "moving" two steps towards any direction, then turning by 60 degrees, and "moving" one more step, the closest BSs at the same frequency band as a given BS can be located. This is illustrated in Fig. 1. Each channel is further divided into three time-frequency slots in a TDMA scheme, resulting in 21 time-frequency slots per BS of Provider 1 and 12 slots per BS of Provider 2. Note that a single time-frequency slot of a given BS can be offered to only one client. Also, the demand of each client is *exactly one slot*.

We consider two different BS deployments, namely, the *uniform deployment*, in which the network of each provider covers the entire city, and the *non-uniform deployment*, in which six BSs (out of 49) of provider 2 are removed. Clients located in the neighborhood of the removed BSs can buy spectral resources only from the provider 1. This is an example of a partial monopoly, in the sense that there are regions in which clients have only the option of connecting to BSs of a single provider.

Providers use the polynomial-based approximation of their payoff function (as described in Section 2) to determine the prices for card clients dynamically, at time instances generated by a Poisson process with a mean rate 0.002 renewals per minute. We assume that both providers offer the same prices to subscribers, which remain fixed through the entire duration of the experiment. This is a reasonable assumption, given the time duration (30500 minutes or 21 days) and scale of these experiments. To penalize an aggressive increase of the transmission power, the providers adopt a pricing scheme that charges the clients proportionally to the transmission power they invest (as described in Section 2). Moreover, the maximum allowable transmission power that a client can invest is 2 Watts.

Client populations There are 5000 clients in total (4400 card users and 600 subscribers), distributed according to a uniform distribution in the simulated region of this small city. In our experiments, the price tolerance threshold (in euros per minute) and target transmission rate (in Mbps) follow a Gaussian distribution. Specifically, we simulated client populations with *normal price tolerance* ($m = 0.15$, $\sigma = 0.0375$) and *high price tolerance* ($m = 0.2$, $\sigma = 0.0375$). We also simulated client populations with a *normal target transmission rate* ($m = 0.1$, $\sigma = 0.01$) and *high target transmission rate* ($m = 0.2$, $\sigma = 0.01$).



(a) (b)

Figure 1: Closest BSs using the same frequency band when the spatial reuse factor is 4 (left plot) and 7 (right plot).

Client demand A client generates a sequence of call requests. The call duration follows a Pareto distribution ($x_s = 3.89$, $a = 4.5$) of mean 5min, while the disconnection period follows a Log-normal distribution ($m = 3.22$, $\sigma = 0.37$) of mean 27min. We assume that during disconnection periods, clients move with pedestrian speed of maximum value 1 m/sec, while they remain stationary during calls. Furthermore, during a call, the client remains connected at the same BS for the entire duration of the call. Fig. 2 shows a snapshot of the network topology. Specifically, it represents the BS deployment and all clients with a call at that particular time instance.

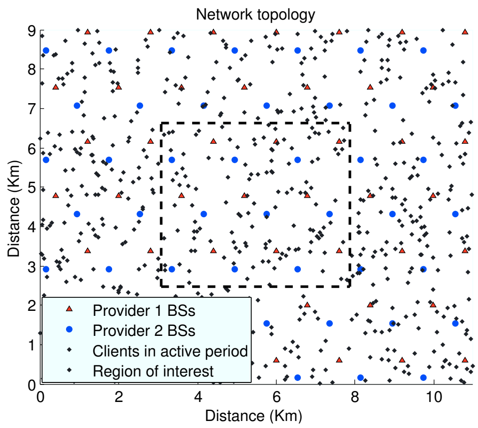


Figure 2: A snapshot of the network topology.

Region of interest To avoid the effect of boundary conditions, each provider takes into consideration only the interactions between BSs and clients located in a small rectangular region, corresponding to the center of the city (marked as "region of interest", the inner

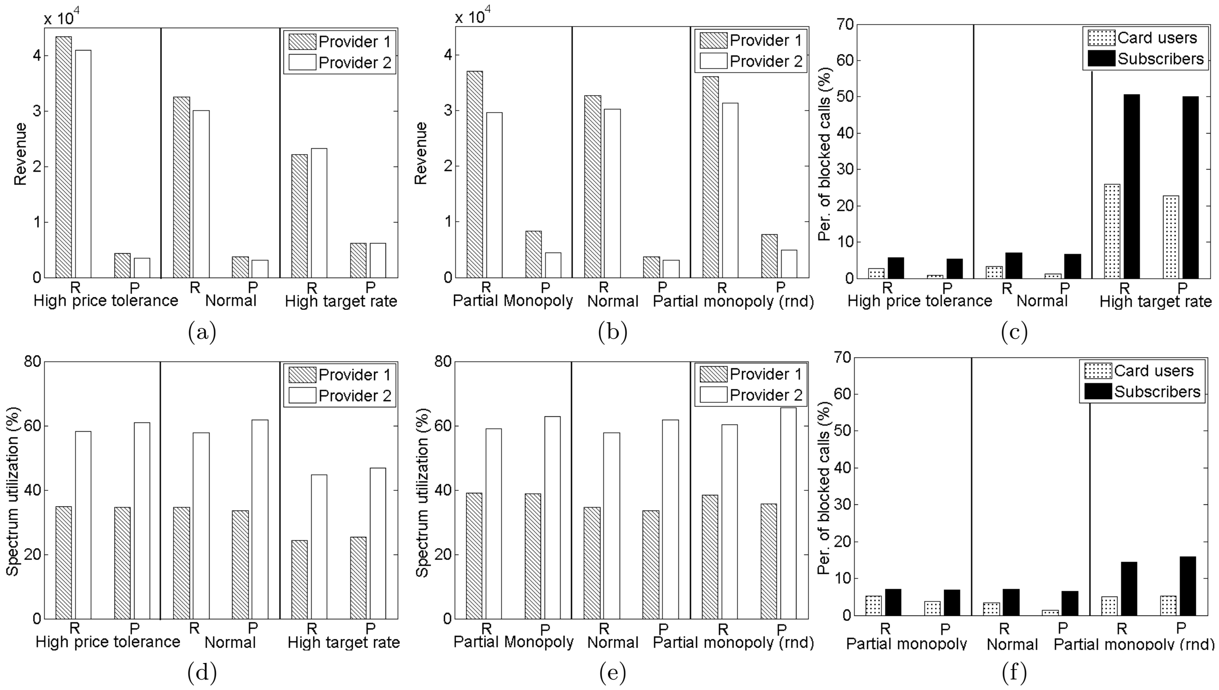


Figure 3: Main results for a cellular-based market. Providers: Revenue (a) and (b). Spectrum utilization (d) and (e). Clients: Percentage of blocked calls (c) and (f). Averages over 10 simulation experiments, each lasting 30500 min.

rectangle shown in Fig. 2). The region of interest includes 9 BSs of each provider. In the non-uniform BS deployment (*partial monopoly*), two BSs of the provider 2 are removed from the region of interest and four BSs are removed from the remaining area. *Only* the BSs located in that region and calls originated from that region are considered in the price adaptation algorithm.

Metrics This analysis will evaluate the impact of client characteristics and preferences, BS distribution (presence of partial monopoly) on the performance of providers and clients.

The performance of a provider is characterized by its revenue and spectrum utilization while the performance of a client is indicated by the percentage of blocked calls. The *revenue* of a provider corresponds to the average total revenue of all BSs in the region of interest that belong to that provider, averaged over all Monte-Carlo runs. The *spectrum utilization* of a BS corresponds to the average percentage of time frequency slots allocated to clients. The *spectrum utilization* of a provider corresponds to the average utilization of all its BSs in the region of interest, averaged over all Monte-Carlo runs. The *percentage of blocked calls of a client* is the ratio of its successful calls over the total number of call requests. Our reported results are average statistics over all clients.

We implemented the simulation platform and this market in Matlab. 10 Monte Carlo runs were performed for each scenario (shown in Table 3). Each scenario simulates a *homogeneous* client population with respect to preference and thresholds. Specifically, "P" scenarios correspond to a price-preference population, while "R" scenarios to a rate preference ones. For each scenario in Table 3, we simulated two client populations, one with price-preference (P) and another with rate-preference (R) (Fig. 3). Note that in partial monopoly (rnd) scenarios, subscribers select randomly their provider. Each run represents the evolution of the market in the microscopic layer and lasts 30500 minutes (including a warm up period of 500 min). Compared to clients, the decision making and updating times of providers occur in longer time scales. The relatively long duration of our simulations is required in order to better observe the evolution of providers and their interaction with clients in this simulated small-city environment. As described in Section 2, the mesoscopic and macroscopic layers enable us to assess the evolution of such markets in even larger spatial and temporal scales.

Table 3: **Description of Scenarios**

Scenario	**Price threshold**	**Rate threshold**	**BS deployment**	**u-map used**
Normal	normal	normal	uniform	yes
High price tolerance	high	normal	uniform	yes
High target rate	normal	high	uniform	yes
Partial monopoly	normal	normal	non-uniform	yes
Partial monopoly (rnd)	normal	normal	non-uniform	no

4.2 Analysis

In general, price preference (P) triggers a more intense competition among providers than rate preference (R). This results in relatively lower prices: fewer users will be blocked due to their price tolerance threshold (Fig. 3 (c)&(f)). Furthermore, in rate preference (R), the revenue is much larger (an order of magnitude) than in price preference (P), in which the competition between providers forces them to keep their prices relatively low (Fig. 3 (a)&(b)). In rate preference, clients tend to buy with a price equal to their maximum price tolerance threshold (in order to increase their transmission rate), while in price preference, clients are more conservative (in that they aim at paying the minimum possible price to achieve the targeted transmission rate). We observed similar trends in scenarios with stationary users that are always "on call" (with a continuous demand for access that lasts during the entire simulation). These results have been omitted due to lack of space.

In the case of increased target rate, as expected, the blocking probability also increases (Fig. 3 (c)). Interestingly, in rate preference, the revenue of providers will decrease. This is due to the fact that, although in rate preference scenarios, clients invest their *maximum transmission power that satisfies the price threshold* in order to achieve the highest possible data rate, for high target rates, fewer clients will achieve their target rate, and therefore, the blocking probability will increase, resulting to a smaller revenue and spectrum utilization.

In price preference scenarios, clients select the least expensive BS (if any) that satisfies their rate and price constraints. As the target rate increases, the price-based selection criterion "deteriorates", since a client will tend to select more frequently the BS that is "closest" to it (i.e., BS with the best channel quality) than the least expensive one (compared to lower target rate scenarios) in order to satisfy the increased data rate requirement. This allows providers to increase their prices, and thus, their revenue. (Fig. 3 (a)). Note that as the target rate increases in price preference scenarios, the BS selection mechanism exhibits more similarities as in rate preference scenarios (i.e., clients tend to choose the BS with the best channel quality). As observed also in rate-preference, the blocking probability is increased, which results to smaller spectrum utilization.

In rate preference, as the price threshold increases, we would expect that the blocking probability decreases, while the spectrum utilization also increases (Fig. 3 (c) and (d)). Interestingly, these changes are small, due to the inter-dependency of the price tolerance threshold of clients and price setting mechanism of providers. The increase of price tolerance threshold allows providers to increase their prices even further. The increase of prices is directly reflected on the increased revenue of providers. Although the blocking probability and spectrum utilization have not changed, the prices are now higher.

As a result of the relatively higher prices in rate preference compared to price preference, the blocking probabilities are larger. Note that this is true only for card users. For subscribers, the prices are the same in both scenarios and remain fixed for the entire duration of the experiment. In addition, in price preference, the higher the price tolerance threshold, the lower the blocking probability.

Card clients have smaller blocking probability than subscribers, since on average, a subscriber is further away from the "best" BS than a card client. This is because a subscriber "belongs" to a provider, and thus, selects a BS from the set of BSs deployed by that provider, while a card client may select a BS from a larger set of BSs that belong to various providers.

In general, provider 1 has a higher spectrum availability (i.e., larger number of time frequency slots) resulting in larger revenue compared to provider 2 and smaller spectrum utilization. Moreover, this is even more prominent in the partial monopoly case, in which the difference in the spectrum availability of the two providers is increased.

In partial monopoly, unlike the case of rate preference, in which the revenue increase is not dramatic, in price preference, the revenue of the monopoly provider is doubled. This is due to the price tolerance threshold and the competition with the other provider (shown in Fig. 3 (b)). Actually, the price-tolerance-threshold is the dominant factor, given that in rate preference, the impact of competition is less prominent since clients select the BS with the best channel quality (and not the lowest price). Note that in monopoly scenarios, there are some regions in which BSs of both providers are present, resulting in a competition. In the region of monopoly, the price setting mechanism of the provider is constrained by the price tolerance threshold, while in the remaining regions, by mainly the competition among providers. The larger the region of a monopoly, the larger the flexibility for that monopoly (provider) to set its price. The competition between providers in the other regions and the tendency of the monopoly provider to increase its price give the opportunity to the other provider to also increase its price, and thus, its revenue. This is an example of cases where partial monopolies provide opportunities to non-monopoly providers to increase their revenue.

The values of u-map indicate that the spectrum availability of provider 1 exceeds the spectrum availability of provider 2. As expected, all subscribers select the provider 1. To evaluate the impact of the u-map, we employ a baseline scenario in which subscribers select a provider randomly (rnd). Compared to the random selection (rnd), the map-based selection exhibits lower average blocking probability for subscribers, both in rate and price preference (Fig. 3 (f)). Clearly a u-map related system can be beneficial to clients. Potentially providers could also take advantage from the reported information about the call arrivals and distributions and user price tolerance threshold. For example, an increased blocking probability in certain areas may alarm providers for further investigation and better capacity planning. In this study, the price adaptation algorithm of providers does not employ any information about clients. It is important to note that the integration of additional knowledge about the population may further improve the performance of the price adaptation mechanism by satisfying the price tolerance threshold of a larger client population. The u-map can provide the information that the more traditional, game-theoretical dynamics require (e.g., well-defined payoff functions for providers). Even more importantly, a network operator could use information collected from such databases to instantiate a market in the mesoscopic and macroscopic layers (as presented in our modelling framework) and analyze its evolution.

5. CONCLUSIONS AND FUTURE WORK

The paper presented a powerful innovative game theoretical framework that models spectrum markets in multiple spatio-temporal scales. It enables a researcher to control the loss of information and "scale up or down" the models of various parameters in order to analyze certain phenomena or trends in the appropriate spatio-temporal scales. Furthermore, it describes a simulation platform that allows researchers to instantiate various paradigms, driven by business cases/scenarios of such markets and analyze their evolution. As an example, it implemented and evaluated a cellular-based market in the microscopic scale. The modelling framework and simulation platform can be extended to instantiate different network and cooperation paradigms (e.g., mesh and cognitive radio networks). For example, a device, e.g., an AP in a mesh network or a primary device in a cognitive radio network, may act at certain times as a provider and at other times as a client, corresponding to two agents. In other cases, providers may form coalitions or bargain with each other or employ jointly power-control and channel allocation.

The amount of available information and its reliability are critical parameters. Our proposed price adaptation algorithms assumed that providers have no knowledge about the client preference and constraints or the payoffs of other providers. It is a part of our on-going research to compare this approach with the other more traditional game-theoretical dynamics (also described in this work) that assume such knowledge. Note that the u-map serves as a "connecting axis" among the various types of dynamics enabling the integration of different types of information in the simulation platform. We plan to incorporate the presence of malicious, mis-configured or non-rational entities in our simulation platform. For example, malicious/mis-configured clients may upload erroneous information on the u-map, while non-rational entities can make "mistakes" (aspects that have been incorporated in our modelling framework).

An important long-term objective of this research is the incorporation of measurements from a real-life network environment in our simulation platform. Specifically, we consider the integration of information collected from a metropolitan-area wireless network, e.g., BSs deployment, empirical-based channel models (e.g., ray-tracing), mobility models, and user traffic traces. This will further enrich the simulation platform by enabling the cross-validation and analysis of various paradigms and spectrum markets in even more realistic settings. The proposed modelling framework sets the fundamentals for enabling the assessment of the evolution of markets of cognitive radio technologies using game-theory and micro-economics.

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